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Addressing buckling of compression members using subdivision of force diagrams

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Abstract

This paper demonstrates how buckling issues can be addressed early in a design process using graphic statics and force diagram modification. It explores how the insertion of a single node in a closed polygon of the force diagram leads to the insertion of a polygon in the form diagram, and how this method can be used to effectively reduce the length of members and significantly increase the buckling capacity of structures.

The research highlights the degrees of freedom of the resulting form diagram, and respectively identifies important subdivision parameters that can be modified to establish a determinate reciprocal relationship between a subdivided force diagram and the corresponding form with no buckling in the compression members. To evaluate the performance of the resulting form, a buckling-adjusted load-path formula is used (Block *et al.* [7]). It is closely investigated how the mentioned subdivision parameters can be chosen to effectively reduce the required amount of materials in the structure.

Keywords: Buckling performance enhancement, force diagram modification, force polygon subdivision, graphic statics, equilibrium modeling.

1. Introduction

The design of slender structures will often be limited by buckling problems, which may force the architects to make undesired changes at a late stage of the design process (Neves *et al.* [12]). The inclusion of buckling concerns makes the evaluation of structural performance more complex, and the concern is often not included in early analysis and optimization. However, when including these concerns the calculated performance of a structure and the optimal layout of members may change significantly (Hemp [9], Neves *et al.* [12], Mazurek *et al.* [11]). Recognizing such concerns late in a design process can lead to delays and additional costs. This paper will show how buckling concerns can be included and addressed early in a design process using graphic statics and force diagram modification.

The reciprocal relationship between form and forces allows a designer to indirectly modify a form by directly modifying the desired distribution of forces. Changes in a force diagram are directly reflected in the corresponding form diagram. Modifying a force diagram while all force polygons are kept closed guarantees that the corresponding form diagram is in both external and internal equilibrium. By keeping the lines of the force polygon that corresponds to the external forces intact, it is further guaranteed that the boundary conditions of the structure are unchanged (Akbarzadeh *et al.* [1]). As the form is not subject to direct modification, such methods can lead to *"feasible designs that are not biased towards known solutions or predefined typologies"* (Lee *et al.* [10]).

Subdivision of force diagrams is one such example of 'force diagram modification'. The method has been developed by Akbarzadeh *et al.* [1, 2] who suggest that it *"can be used as a strategy in dealing with buckling in spatial funicular structures".* The buckling capacity of a compression member depends greatly on the length of the member. Hence, if the structural capacity of a truss member in compression is governed by buckling, shortening the member will increase its strength significantly (Eurocode 3 [8]). As subdivision of force diagram can lead to shortening of members the potential of the method as a way to address buckling is interesting.

For simplicity reasons, the method is only considered for 2D graphic statics. It is, however, suggested to expand the analysis to 3D graphic statics. This paper uses *Bow's Interval Notation* (See Baker *et al.* [4] and Allen & Zalewski [3] for more details) to more intuitively describe the relation between form and force diagrams.

First, the properties of *Node-Insertion Subdivision* are covered. This is followed by an example showing how the method can be applied on a structure with known buckling problems. Then, the buckling adjusted performances of structures are evaluated and it is shown how the choice of subdivision parameters greatly influences the performance of the resulting structures.

2. Node-Insertion Subdivision

In this section, the geometric properties and degrees of freedom of node-insertion subdivision are explored.

This paper will focus on only one type of subdivision, which will be referred to as *Node-Insertion Subdivision*: In a force diagram, a *Subdivision Node* is inserted. Every existing node of a desired closed force polygon is then connected to the new node, essentially triangulating the closed force polygon. A special case of node-insertion subdivision is when the new node is placed in the barycenter of the force polygon. This case is called *Barycentric Subdivision* (Petersen [13]) and has been used by Akbarzadeh *et al.* [1] in the design and form finding of compression-only structures. Node-Insertion Subdivision has been chosen for this research due to its well-defined and simple properties and promising resulting structures.

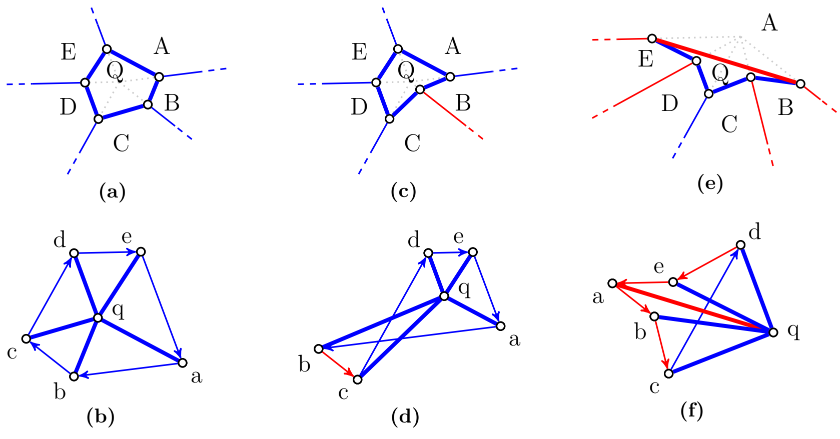


Figure 1: Subdivision of a node *A-B-C-D-E* (The node may be part of a larger truss not shown in the figure) by inserting a node *q* in the force diagram. In all force diagrams, the initial force polygon is shown with arrows and new members highlighted with thick stroke. Members in tension and compression are shown with red and blue respectively. (a) & (b): form and subdivided force diagram of a compression only node; (c) & (d): form and subdivided force diagram of compression/tension-combined node. Member *C-B* is in tension; (e) & (f): form and subdivided force diagram of a compression/tension-combined node with two neighboring members of the initial form meeting in a reflex angle.

The form and force diagrams of graphic statics are each other's topological duals. This means that a polygon in the one diagram corresponds to a node in the other; inserting a node in the force diagram results in the insertion of a polygon in the form diagram. Triangulating a closed polygon with *N* sides in the force diagram corresponds to replacing a node of valency *N* in the form diagram with *N* nodes of valency 3. In Figure 1, three different cases of subdivision are shown.

The corresponding form of a subdivided force diagram is characterized by shortening of original members. This property can be used to actively address buckling issues.

2.1. Subdivision parameters

The location of an inserted *Subdivision Node* in a 2D force diagram has two degrees of freedom (DOF). The corresponding form is reciprocal, but not determinate and has one DOF: the inserted polygon has to be scaled. In figure 2 the process of creating the form, corresponding to a subdivided force diagram, is shown. A point defined by a parameter s_0 along an arbitrary member is chosen as the anchor point. From this point, a line parallel to the corresponding line in the subdivided force diagram is drawn and its intersection point with a neighboring original member is found. It is seen that all intersection points between new and old members are dependent. In Figure 3, different choices of the scale of the subdivision polygon are shown.

Hence, subdivision of force polygons has three DOF. One can choose a subdivision point on the 2D plane of the force diagram and find the corresponding form with one geometric DOF. It is also possible to select three constraints in the form diagram and derive the corresponding subdivided force diagram. This will be used in the following example.

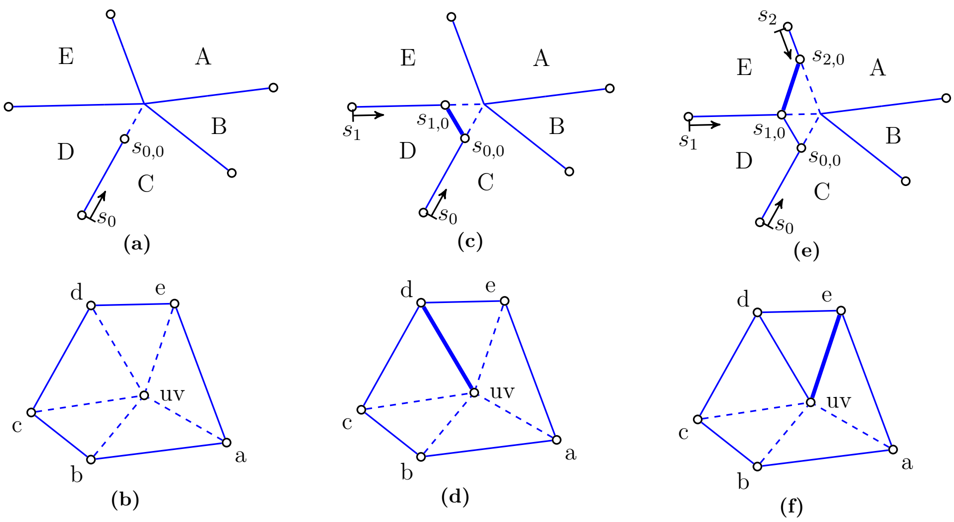


Figure 2: Process of extracting the form corresponding to a subdivided force diagram; top: form diagrams, and bottom: force diagrams. All members are in compression (shown with blue). Members with thick stroke show the considered member at each step. The parameters $s_{j,}$ define the point along a member at which the inserted polygon intersects.

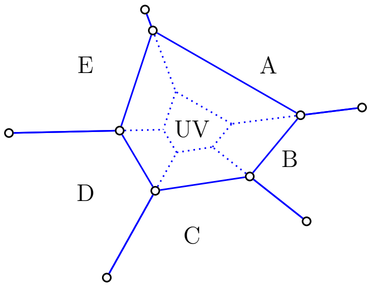


Figure 3: Different scales of subdivision polygon

3. Addressing known buckling problems in structures.

This chapter will show how subdivision of force diagrams can be used to address buckling issues in a structure with predefined cross section and material properties that has buckling problems.

The following calculations are based on Eurocode 3 [8]. The material properties are as follows: steel S235 with yield strength f_y=235MPa (the reduction factors are ignored) and module of elasticity E=210GPa. It is assumed that thin-walled circular cross sections are used with cross section area A=2200mm^2 and second moment of area I=2.749·10^{6}mm^4. The imperfection factor for compression members is α=0.49.

Now consider the structure shown in Figure 4. The structure is symmetric and only the left half of the form will be considered. Members *E-1* and *E-3* are the only members in compression. In Table 1, the actual and required length properties are shown. It is seen that the compression members does not satisfy the buckling criteria. For the criteria to be satisfied, the length of the members must be maximum 4.15m and 4.65m, respectively.

It is now used that subdivision of force diagrams has three DOF. In the form diagram, two of the three necessary parameters are chosen: the subdivision polygon must intersect the two original compression members such that they both satisfy the buckling criteria. A new member is inserted spanning between these two intersection points. The new member will be a part of the inserted polygon resulting from subdivision and will be referred to as member *E-Q* (See Figure 5). All members are now of equal length L=3.98m. The form still has one degree of freedom. In the force diagram in Figure 5, the direction of the force line is shown. The subdivision node must be placed on this line.

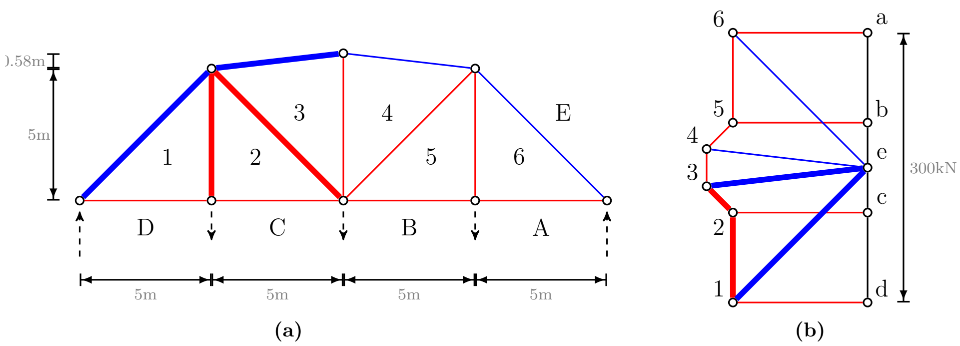


Figure 4: Form (a) and force (b) diagram of truss structures. Node considered for subdivision shown with thick stroke. Members in tension and compression shown with red and blue respectively.

Table 1: Load carrying properties of compression members in Figure 4. P is the internal force in considered member, L is the length of the member, λ is the dimensionless slenderness, χ is the reduction factor for buckling, $N_{b,Rk}$ is the buckling resistance and the utilization degree must be ≤1 for the member to have sufficient strength. Line with member name without star shows calculations for the actual length of the member and member name with * is for the required length of the member.

Member	P [kN]	L [m]	λ	χ	$N_{b,Rk}$ [kN]	utilization
E-1	**212**	**7.07**	2.13	0.18	91	**2.33**
*E-1**	-	**4.15**	1.25	0.41	212	**1.00**
E-3	**180**	**5.03**	1.52	0.31	160	**1.13**
*E-3**	-	**4.65**	1.40	0.35	181	**1.00**

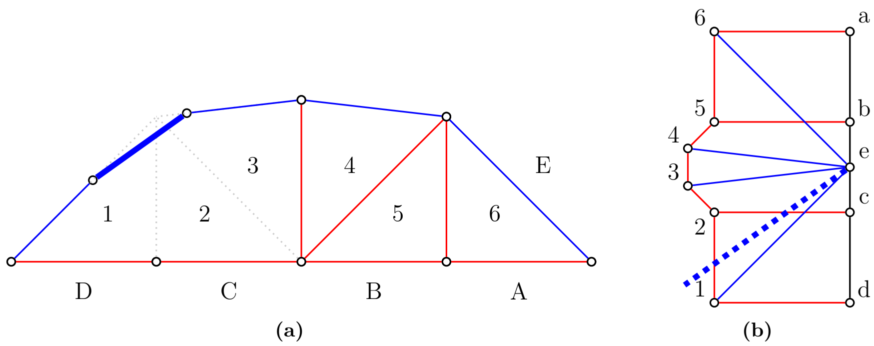


Figure 5: Subdivision polygon member, highlighted with thick stroke, inserted in form diagram (a). The direction of line shown in force diagram (b).

Table 2: Load carrying properties of compression members in Figure 5. All members are of equal length. Hence, they have equal buckling properties.

Member	P [kN]	L [m]	λ	χ	$N_{b,Rk}$ [kN]	utilization
E-1	212	3.98	1.20	0.43	225	0.94
E-3	180	-	-	-	-	0.80
E-Q	220	-	-	-	-	0.98

In Table 2 it is seen that all members have a buckling resistance of 225kN. This further restricts the location of the subdivision node in the force diagram. The subdivision node must be within the shown domain of the highlighted line in the force diagram of Figure 5.

Another constraint is required. For visual reasons, it is desired that the member Q_1-3 is vertical (many other constraints could be used). In figure 6, the member is highlighted in both the form and force diagram. The location of the subdivision node q is now determined and the process is finished. In table 2 it is seen that all compression members satisfy the buckling criteria.

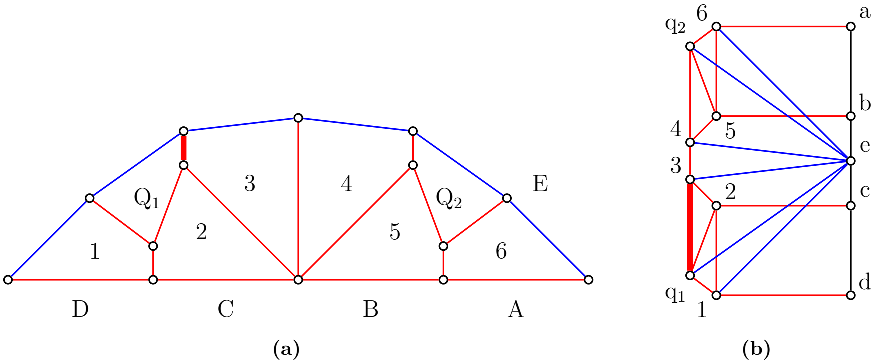


Figure 6: Final form and force diagram in which no members have buckling problems. Member Q_1-3 highligted with thick stroke.

4. Enhancing buckling adjusted performance

This section will show how the choice of subdivision parameters greatly affects the buckling adjusted performance of a structure. A buckling adjusted load path formula will be used.

The load path of a structure is often considered directly proportional to the material volume requirement and is frequently used in optimization and evaluation of structures (Beghini *et al.* [5,6], Baker *et al.* [4]). However, the load path of a structure does not include buckling concerns. This problem has briefly been addressed by Block *et al.* [7] in the development of the graphic statics tool *TrussPath*. The load path formula includes the reduction factor χ for buckling and is

$$\sum_{i}^{N} L_i P_i = \sum_{i}^{N_t} L_{t,i} P_{t,i} + \sum_{i}^{N_c} L_{c,i} P_{c,i} \chi_i^{-1} \tag{1}$$

where N is the total number of members, L_i and P_i are the length and absolute value of the internal force of a member, respectively. The product of L_i and P_i is the *Load Path* of the member such that ΣLP is the *Total Load Path*. The subscripts t and c refer to members in tension and compression, respectively. The buckling reduction factor χ is based on Eurocode 3 [8] and is a function of the length of the member L_i, the imperfection factor α and cross section and material properties. The total buckling adjusted load path of a structure can be assumed to be directly proportional to the buckling adjusted material requirement.

In this section all members are assigned a fictional circular cross section with constant radius. The use of other assumptions, such as constant thickness or a constant ratio between radius and thickness is suggested for further research.

In the following example, it is shown how different choices of subdivision parameters can yield different performance results, even if the regular load path (χ=1) does not show any difference. For all structures, the following material properties are assumed: Material strength is f_y=235MPa (any reduction factors are ignored) and module of elasticity E=210GPa. The imperfection factor of compression members is α=0.49. Radius of members is r=40mm

Consider the compression-only form and force diagram in Figure 7. It is assumed that the structure is part of a larger structure such that all nodes are constrained against movement. Using the before-mentioned assumptions, the buckling adjusted load path is found to be **Σ***LP*=16.8MNm. If buckling is not considered (χ=1) the load path is **Σ***LP*=3.3MNm.

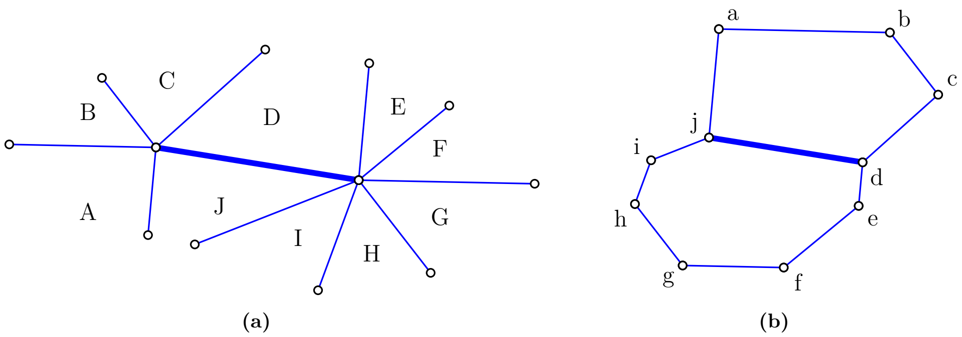


Figure 7: (a) Form and (b) force diagram of compression only structure. Length and internal force of highlighted member: 7m and 100kN. Regular Load path = 3.3MNm. Buckling Adjusted = 16.8MNm

Now assume that the force diagram is subdivided by inserting a node in each of the closed force polygons. Each subdivision introduces a polygon in the form diagram, each of which has one degree of freedom: The scale. In Figure 8 different scalings of the polygons are presented. It is seen, that for a known force diagram the performance of the resulting structure greatly depends on the choice of scale parameters. The best performing of the three shown solutions has a buckling adjusted load path of ΣLP=5.6MNm. This is a reduction of 67%, compared to the initial solution. Note that the regular load path is still ΣLP=3.3MNm.

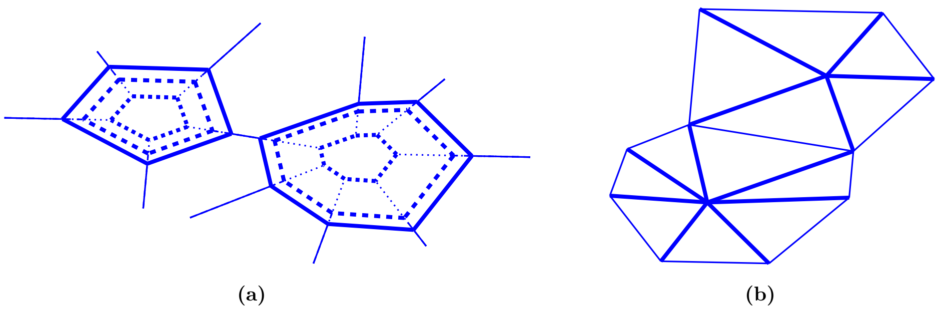


Figure 8: (a) Different scaling of polygons in form diagram. (b) Subdivided force diagram shown in. Buckling adjusted load path: Dashed lines = 5.6MNm, Dotted lines= 8.2MNm and Fully drawn lines=6.2MNm. New members highlighted with bold stroke.

In Figure 10 a similar example is shown. Here the start and end point of the member connecting the two initial nodes are kept constant and a subdivision points can be freely chosen in the force diagram. Again it is seen that the performance of the resulting structure greatly depends on the choice of subdivision parameters. The two shown structures have a buckling adjusted load path of 5.6MNm and 7.9MNm respectively. Both solutions result in a reduction of load path compared to the initial structure.

As seen in the previous examples, subdivision of force diagrams can be used to design structures that theoretically require less material than the structure from which they emerge. More examples are shown in Figure 9 where also recursively subdivided structures are shown.

The choice of parameters greatly affects to which degree the material requirement is lowered, some parameter values may even lead to an increase. In a computational implementation, multivariable optimization techniques can be used to find the best performing set of parameter values.

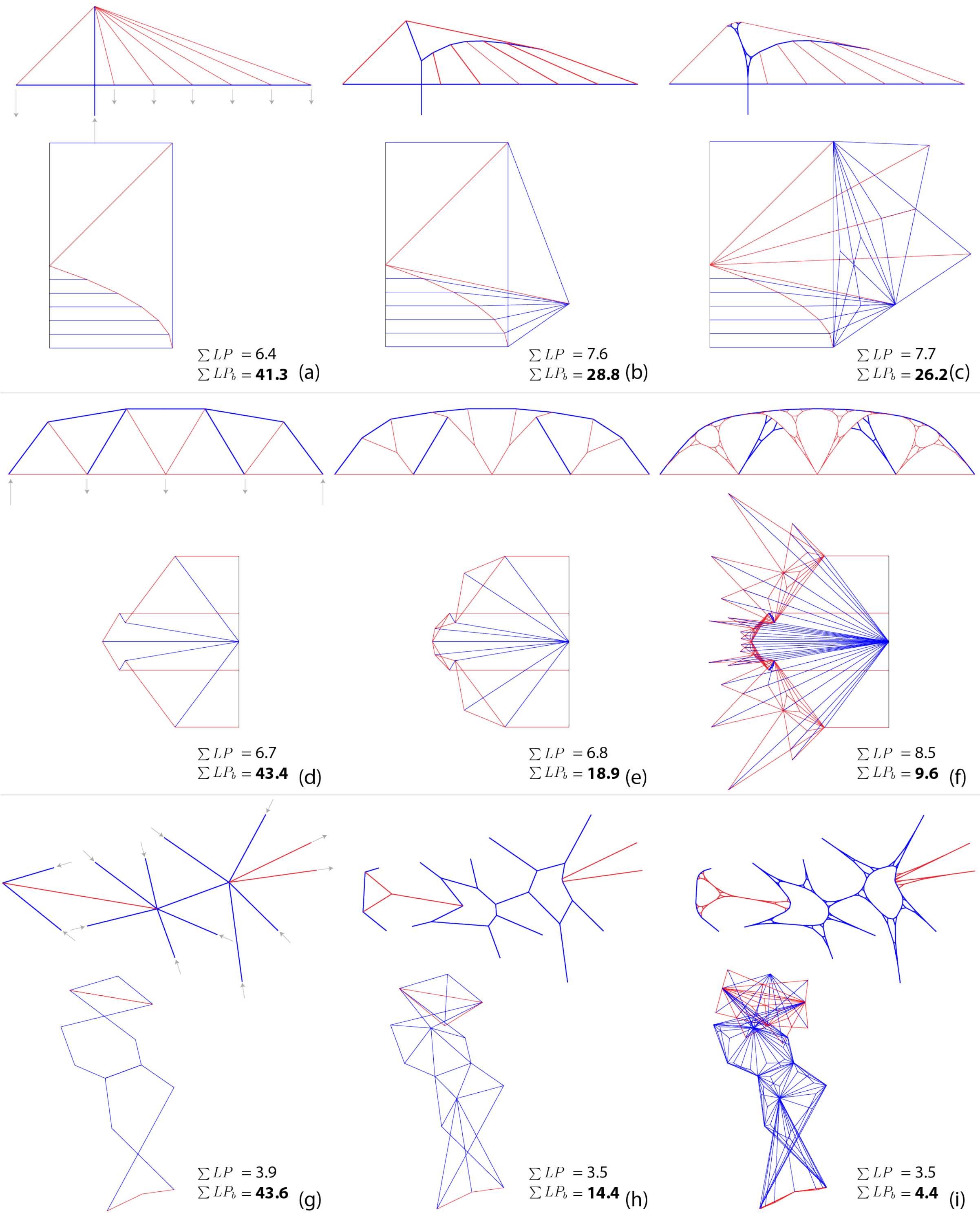
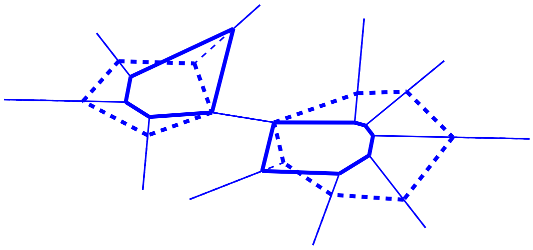
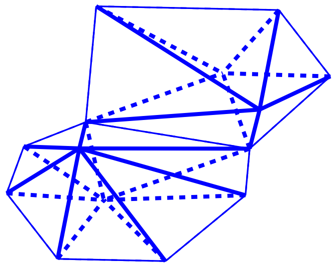

$\sum LP$ = 6.4
$\sum LP_b$ = **41.3** (a)
$\sum LP$ = 7.6
$\sum LP_b$ = **28.8** (b)
$\sum LP$ = 7.7
$\sum LP_b$ = **26.2** (c)
$\sum LP$ = 6.7
$\sum LP_b$ = **43.4** (d)
$\sum LP$ = 6.8
$\sum LP_b$ = **18.9** (e)
$\sum LP$ = 8.5
$\sum LP_b$ = **9.6** (f)
$\sum LP$ = 3.9
$\sum LP_b$ = **43.6** (g)
$\sum LP$ = 3.5
$\sum LP_b$ = **14.4** (h)
$\sum LP$ = 3.5
$\sum LP_b$ = **4.4** (i)

Figure 9 (previous page): Three different structures subdivided. In each subfigure the form and force diagram are in top and bottom respectively. In form diagrams compression members are highlighted with red. The initial form diagrams (left figure in each row) are each 40m from left to right and initial force diagrams are 100kN from top to bottom. In each row the initial structure is shown along with a regular subdivision. In the final subfigure of each row the result of recursive subdivision is shown. Regular and buckling adjusted load path values shown in each subfigure in MNm with regular and bold text respectively.



(a)



(b)

Figure 10: Different choices of subdivision node location. (a) Form diagram. (b) Subdivided force diagram. Buckling adjusted load path: Dashed lines = 5.6MNm. Fully drawn lines= 7.9MNm.

Even though the theoretical material usage is lowered, the complexity of structure does, however, increase and the resulting structures are not necessarily stable. It is only guaranteed that the structures are in equilibrium for the given load case. Due to these factors, the theoretical performance enhancement does not necessarily reflect a reduction of the final cost of the structure (Mazurek *et al.* [11]). Rather, the evaluation of structural performance can be used to assess how different subdivided forms perform against one another, and the method can be used as a tool to freely explore possible well-performing design possibilities.

7. Conclusion and Discussion

This research showed that node-insertion subdivision of force diagrams can be used to effectively shorten members to satisfy the buckling criteria. The considered subdivision method has three degrees of freedom. These three parameters can be selected carefully to enhance the buckling performance of a structure. Two cases were considered:

1) a structure with known buckling problems: by determining the maximum allowable length of compression members the subdivision parameters may be chosen to ensure that no members buckle.

2) Performance assessment: A buckling adjusted load path as suggested by Block *et al.* [7] is used. It was shown that for the design of structures with slender members, subdivision of force diagrams can be used to effectively reduce the buckling adjusted load path. The selection of subdivision parameters greatly affects the buckling adjusted performance of resulting structures.

Due to the increased complexity of structures the method presented is not necessarily effective as an optimization tool. Further, the resulting structures are visually very different than the structures from which they emerge. Hence, the method presented should be considered as a way to freely explore and rationally select possible design options rather than a tool to solve specific problems.

It is suggested for further research to expand the findings to 3D graphic statics. It is further suggested to research how similar force diagram modification methods can be used, such as the insertion of multiple nodes. Lastly, it is suggested to research how structures can be subdivided recursively in a controlled manner to effectively addresses buckling.

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