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Three-dimensional graphic statics: Initial explorations with polyhedral form and force diagrams

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Abstract

This article investigates how reciprocal form and force polyhedrons can be used to develop procedures for the design of three-dimensional trusses and funicular structures, analogous to the well-known techniques of graphic statics for two-dimensional structural systems. It demonstrates how global equilibrium of a system of forces can be established by constructing a closed force polyhedron, if the forces can be replaced by a resultant force alone, without a resultant couple. It also describes the three-dimensional equivalent of the "closing string," which is the basis in graphic statics for the construction of funicular solutions for given loads and support locations. Furthermore, it provides a procedure for constructing a constrained funicular form for a simple, determinate boundary condition. Finally, it discusses some of the difficulties involved with similar constructions and procedures for non-concurrent forces and in particular with those systems of forces that can only be replaced by a resultant force and couple.

Keywords

constrained reciprocal diagrams, graphic statics procedures, Rankine's principle of equilibrium of polyhedral frames, three-dimensional graphic statics

Introduction

Graphic statics is a method for finding efficient structural forms using geometric operations. Traditional (two-dimensional (2D)) graphic statics includes a collection of geometric construction techniques and procedures that have been used, researched, and developed since the late 18th century. Examples of these procedures can be found in previous works.[1–10]

The procedures of 2D graphic statics are based on the 2D reciprocal relationship of form and force diagrams. This reciprocity provides the designers an unprecedented, intuitive control in the design of complex, yet efficient, structural forms and their internal force distribution at initial stages of the design process using only geometric constructions. However, form finding with graphic statics is primarily 2D. The capability of these 2D methods in dealing with three-dimensional (3D) problems is quite limited.

In this article, we investigate the possibility of using reciprocal form and force polyhedrons, rather than form and force polygons, as the basis for a form-finding (and analysis) methodology for spatial funicular structures that is analogous to the methods of graphic statics for 2D structural systems (Figure 1). This investigation is a continuation of previous work by the authors in which they illustrated and clarified Rankine's[11] "Principle of Equilibrium of Polyhedral Frames" (Figure 2) and showed how polyhedral form and force diagrams could be used to explore the geometry of both determinate and indeterminate spatial systems of forces.[12,13]

In section "Global equilibrium of 2D system of forces using funicular constructions," we identify the main steps of the well-known geometric procedure for establishing

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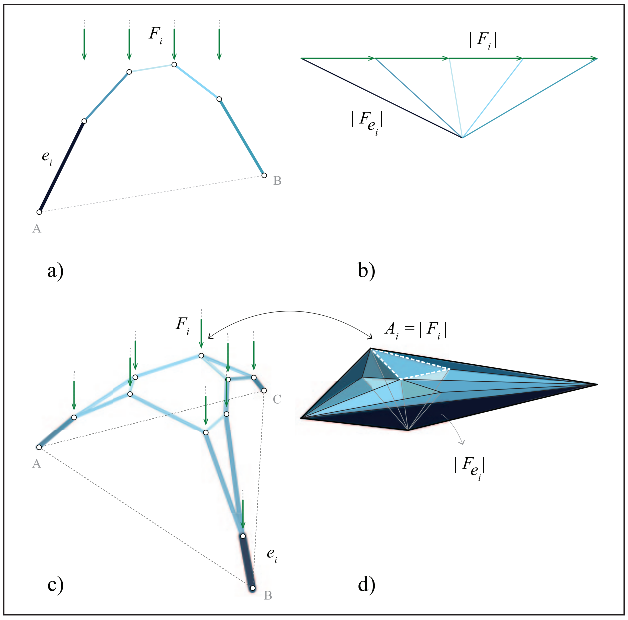

Figure 1. 2D versus 3D funicular solutions and their corresponding force diagrams: (a) a 2D funicular solution constrained to the given boundary conditions, the result of using 2D graphic statics; (b) a force diagram representing the magnitude of the forces in the 2D funicular form; (c) three-dimensional funicular form resulted from the methods of 3D graphic statics proposed in this article; and (d) the force polyhedron representing the magnitude of the forces in its corresponding form.

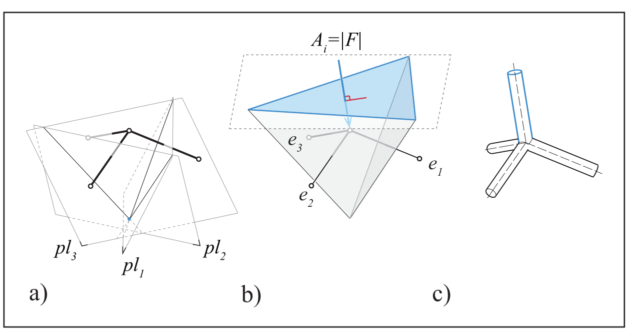

Figure 2. Rankine's principle of the equilibrium of polyhedral frames: (a) planes drawn perpendicular to the bars of a spatial frame constitute an open polyhedron diverging from a point, (b) the closing face of that polyhedron represent the direction (normal to the face) and the magnitude of the force (area of the face) that keeps the internal forces of the bars in equilibrium, and (c) tubular representation of the forces in each bar corresponding to the areas of the reciprocal polyhedron.[12]

global equilibrium of a 2D system of forces using the funicular polygon. In section "Global equilibrium of 3D system of forces using funicular constructions," we describe an equivalent procedure using reciprocal polyhedrons for two specific types of spatial systems of forces: concurrent forces and parallel forces. In section "Funicular form finding for a given boundary condition and applied loads," we present a procedure for constructing a constrained funicular form for a simple, determinate boundary condition. In section "Discussion and future work," we discuss the difficulties involved in dealing with more complex boundary conditions and non-concurrent loads.

Global equilibrium of 2D system of forces using funicular constructions

In traditional (2D) graphic statics, the edges of the form and the force diagrams are typically drawn parallel to each other. In this article, we draw the edges of the 2D form and force diagrams perpendicular to each other to be consistent with the 3D version of graphic statics that is based on the reciprocal relationship between the edges of the form and the faces of the force diagram. This relationship is meaningful only if the corresponding members are perpendicular.

To find the global equilibrium for a 2D system of forces, we need to find the magnitude, direction, and the line of action of the resultant of the forces. Adding a force to the system which is equal in magnitude and opposite in direction with the resultant globally equilibrates the system. This added force is sometimes called "anti-resultant," since it has the same magnitude as the resultant but opposite direction. The global equilibrium of a 2D set of forces can be translated geometrically into the following conditions:

- The force polygon of the system must be closed and the sense going around it must be continuous (translation equilibrium);
- The funicular polygon constructed on the line of action of the forces must be closed (rotational equilibrium).[5]

The following paragraphs explain the process of finding global equilibrium for the given set of forces of Figure 3(a).

Constructing a closed force polygon

In the 2D system of forces of Figure 3(a), drawing the force vectors with their lengths corresponding to their magnitudes and perpendicular to their line of action, successively at the end of each other, constructs an open force polygon (Figure 3(b)). This open polygon can be closed by connecting the start point to its end point. The closing edge of the force polygon (blue) represents the magnitude and the direction of the resultant force that can substitute the forces in the system, and therefore, it can keep the system in global equilibrium if applied in the reversed direction.

Constructing a closed funicular polygon

The force polygon can be decomposed to an arbitrary point p_{trial}, and consequently, the funicular polygon can be

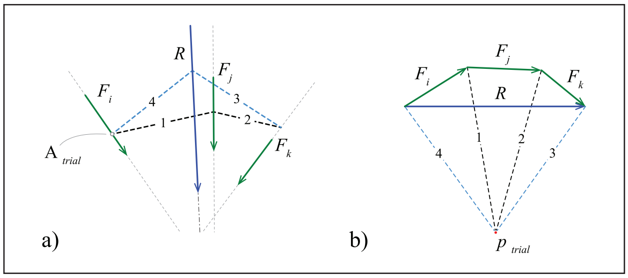

Figure 3. Finding the resultant force using funicular construction for forces F_i to F_k : (a) the force system including the funicular polygon and the line of action of the resultant and (b) the corresponding force polygon closed by the resultant edge.

constructed using trial construction[5] (Figure 3(b)). The vertices of the funicular polygon (dashed) lie on the line of action of the forces in the system, and each edge of the funicular polygon is perpendicular to its corresponding edge of a triangle in the decomposed force polygon (Figure 3(a) and (b)). The construction of the funicular polygon defines the location of the (anti-)resultant force; for instance, the edges 3 and 4 of the funicular polygon intersect at a point where the line of action of the resultant force should be applied (Figure 3(a)). Therefore, using geometric constructions, it is possible to find the magnitude, direction, and the location of the line of action of a force that globally equilibrates the system.

Global equilibrium of 3D system of forces using funicular constructions

It is possible to find the global equilibrium for 3D system of forces using only geometric constructions; the funicular polygon and the force polygon have their equivalent in 3D graphic statics as the *funicular polyhedron* and the *force polyhedron*. Constructing the funicular polyhedron and the force polyhedron for a system of forces in 3D requires further explanation of the possible combination of the forces in 3D funicular, polyhedral constructions.

Loading conditions in 3D reciprocal polyhedra

The funicular, polyhedral construction is based on "The Principle of Equilibrium of Polyhedral Frames."[11–13] According to this principle, both form and force polyhedrons consist of planar faces. Therefore, a 3D system of forces described by this principle includes a group of planes, the intersections of which represent the direction of the applied loads. Figure 4 illustrates various allowed configurations of 3D force systems including parallel, concurrent, and non-concurrent forces.

In this article, we provide procedures to find the global equilibrium by constructing force polyhedron and funicular

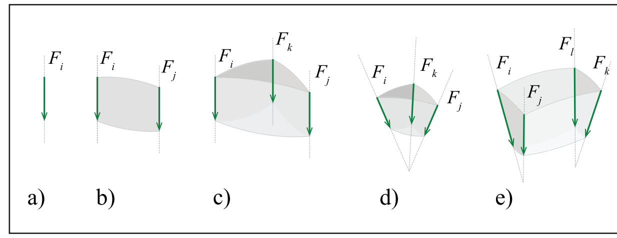

Figure 4. (a) Single, (b) coplanar, (c) parallel, (d) concurrent, and (e) non-concurrent system of applied loads in 3D reciprocal polyhedra.

polyhedron for parallel and concurrent forces of Figure 4(c) and (d). To find the global equilibrium of the non-concurrent system of forces (Figure 4(e)) and the combination of all the possible loading cases, please visit the discussions in section "Discussions and future works."

Conditions of equilibrium in 3D

Equivalent to the 2D case, to find the equilibrium of a 3D system of forces, it suffices to show that the system has a *closed force polyhedron* and a *closed funicular polyhedron*. Therefore, the following steps are necessary:

- Constructing a closed force polyhedron for the applied loads;
- Decomposing the force polyhedron to a group of force tetrahedrons;
- Constructing funicular polyhedron using the decomposed force polyhedron;
- Finding the location of the line of action of the (anti-)resultant to equilibrate the system.

Similar to 2D, an open force polyhedron can be closed by adding an additional force with a face perpendicular to the direction of the (anti-)resultant. The magnitude and the line of action of this force can be found through funicular, polyhedral construction.

Concurrent forces

In a previous study, Akbarzadeh et al.[14] showed the construction of global equilibrium for a given boundary conditions and simple example where the resultant face was constrained to the given boundary conditions. However, their process of finding the global equilibrium did not include the construction of a force polyhedron corresponding to the applied loads, its decomposition, and the funicular polyhedral construction to find the line of action of the resultant. In fact, the line of action of the resultant was found separately and fed to the process. To generalize and therefore complete their proposition, the following sections will explain the process of finding the equilibrium

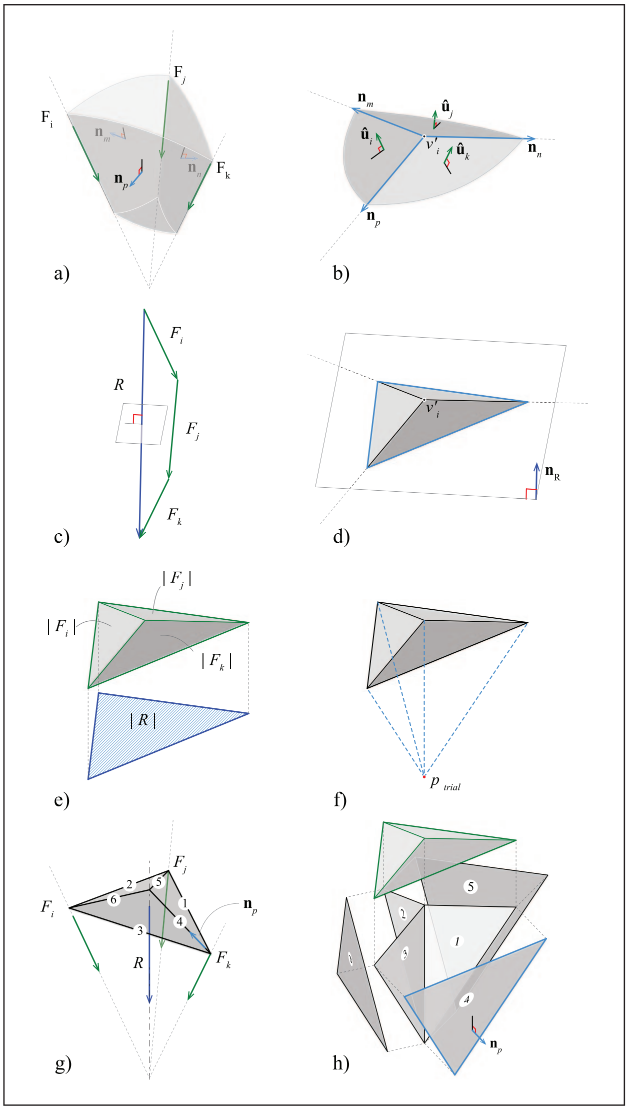

Figure 5. (a) Concurrent set of forces, (b) the construction of the open force polyhedron, (c) the direction of the resultant and the resultant plane, (d) the intersection of the resultant plane and the faces of the open force polyhedron, (e) the closed force polyhedron, (f) decomposing the force polyhedron, (g) funicular polyhedron, and (h) the faces of the force polyhedron corresponding to the edges of the funicular polyhedron.

using polyhedral construction for a concurrent and a parallel system of forces in 3D.

Figure 5 summarizes all the necessary steps to find the equilibrium condition for a concurrent system of forces in 3D, using funicular, polyhedral construction. It therefore describes the procedures to find the magnitude, direction, and the line of action of the resultant force to keep this system in equilibrium.

Constructing a closed force polyhedron. Figure 5(a) illustrates a system of concurrent forces F_i, F_j, and F_k. To construct the force polyhedron, start from an arbitrary point v_i' in 3D and draw lines perpendicular to the faces shared by the forces in the concurrent system (with the directions $\mathbf{n}_m$, $\mathbf{n}_n$, and $\mathbf{n}_p$). This creates an open tetrahedron consisting of three open faces diverging from the point v_i'. Each open face of this open tetrahedron is perpendicular to an applied force in the system (Figure 5(b)). To close this open tetrahedron, add a face perpendicular to the direction of the resultant R by simply intersecting the faces with a plane perpendicular to the resultant that is called the resultant plane (Figure 5(c) and (d)).

The added face closes the force polyhedron and defines the scale and the edges of the triangular faces whose areas correspond to the magnitude of the force F_{i-k}. Consequently, the faces of the closed force polyhedron are perpendicular to the applied forces and their resultant, and the area of each face represents the magnitude of the corresponding force in the system (Figure 5(e)).

Decomposing the force polyhedron. In 2D, a force polygon can be decomposed into a group of force triangles by connecting its vertices to an arbitrary chosen point p_{trial} (Figure 6(a)); the decomposed force polygon of Figure 6(a) includes small force triangles with an edge corresponding to an applied load representing *nodal* equilibrium (Figure 6(b)) and a force triangle that includes the resultant force representing *global* equilibrium (Figure 6(c)).

Similarly, a force polyhedron can be decomposed into a group of force tetrahedrons by connecting its vertices to an arbitrary point p_{trial} (Figure 5(f)); the decomposed force polyhedron, therefore, includes small tetrahedral cells with a face corresponding to an applied load (Figure 6(e)) and a bigger tetrahedron with a face corresponding to the resultant force (Figure 6(f)).

Constructing a funicular polyhedron. Once the force polyhedron is constructed and decomposed into tetrahedral cells, a funicular polyhedron can be constructed accordingly. The construction process shown in Figure 5(g) and (h) is as follows:

- Pick an arbitrary point on the line of action of the applied load F_k.
- From the chosen point, draw a line which is perpendicular to face 1 and intersects the line of action of F_j.
- From the recently found intersection point, draw a line which is perpendicular to face 2 of the force polyhedron intersecting the line of action of the force F_i.
- Drawing a line from the latest intersection on the line of action of F_i perpendicular to face 3 intersects the line of action of F_k exactly at the starting point.
- The lines drawn from the intersection points perpendicular to faces 4, 5, and 6 intersect each other

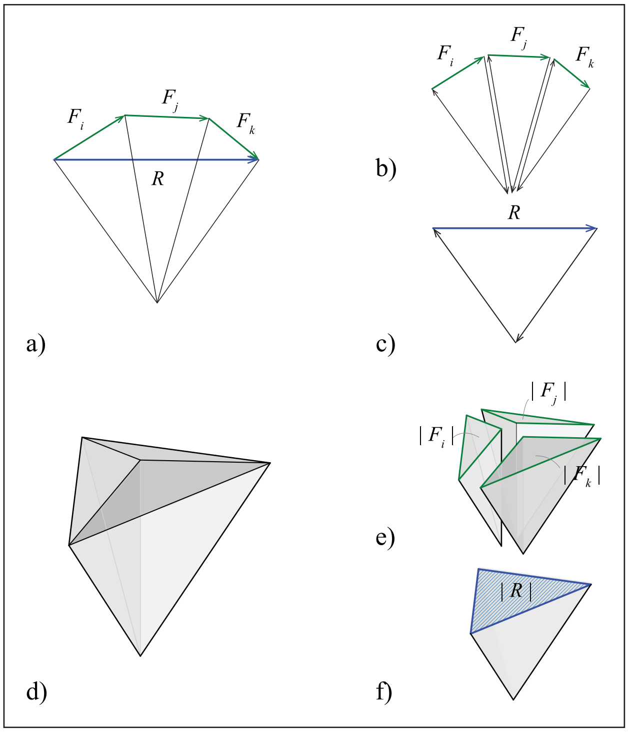

Figure 6. (a) A decomposed force polygon, (b) the force triangles corresponding to the applied loads representing nodal equilibrium, (c) force triangle corresponding to the resultant force representing global equilibrium, (d) a decomposed force polyhedron, (e) the force tetrahedrons corresponding to the applied loads (nodal equilibrium), and (f) the force tetrahedron corresponding to the resultant force (global equilibrium).

and complete the funicular polyhedron. The point of intersection of these lines also defines the location of the line of action of the resultant force R.

Therefore, through geometric operations, it is possible to find the direction, magnitude, and the line of action of the resultant for a concurrent system of forces in 3D. If applied in the opposite direction, this force can keep the system of concurrent forces in equilibrium.

Parallel forces

The system of parallel forces in 3D can be considered as a special case of concurrent forces were the forces intersect at infinity. Figure 7(a)–(j) describes the geometric steps to find the equilibrium of a parallel system of forces using funicular polyhedral construction. According to "The Principle of Equilibrium of Polyhedral Frames" and the previous example, the faces of the force polyhedron are perpendicular to the direction of the forces in the system. Therefore, the faces of the force polyhedron corresponding to the parallel system of forces of Figure 7(a) are coplanar (Figure 7(b)).

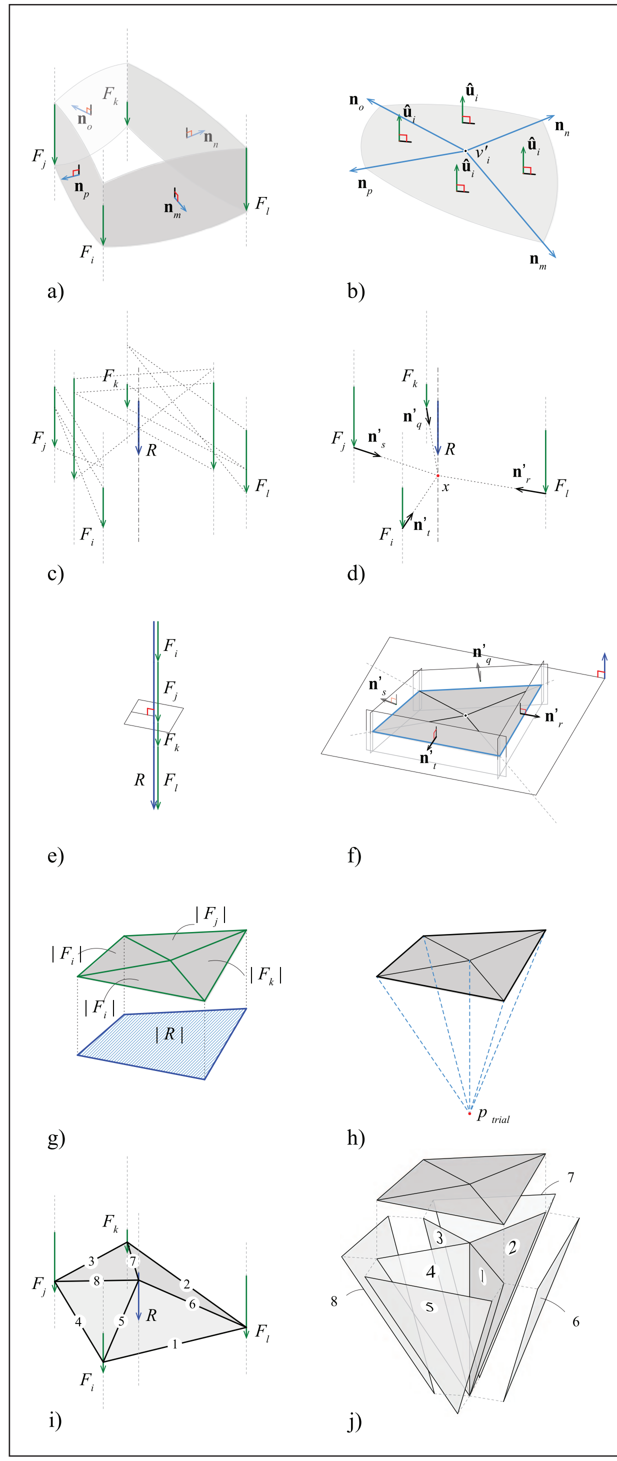

Figure 7. (a) Parallel set of applied loads, (b) open force polyhedron with coplanar faces, (c) finding the location of the resultant using crossing method, (d) connecting the applied loads and the resultant by picking an arbitrary point on the resultant, (e) finding the resultant plane, (f) finding the edges of the force polyhedron, (g) the faces of the closed force polyhedron, (h) force polyhedron decomposition, (i) funicular polyhedron, and (j) the construction order of the funicular polyhedron.

Constructing a closed force polyhedron. To find the edges of the force polyhedron, we need to find the location of the resultant force prior to the funicular construction. The location of the resultant can be found geometrically using the crossing method, or through two funicular constructions on a plane perpendicular to the applied forces.[5] It can also be calculated as the weighted barycenter of the applied loads.

The parallel forces intersect at infinity with the line of action of the resultant. Therefore, the projection of this intersection on the resultant plane includes lines passing through the applied forces and intersecting with the line of action of the resultant at the point x (Figure 7(d)).

These lines are named with their directions as $\mathbf{n}'_{r-q}$. Planes drawn at the edges of the open force polyhedron of Figure 7(b), perpendicular to the directions $\mathbf{n}'_{r-q}$, intersect with the resultant plane. These intersections define the edges of the force polyhedron. Note that the volume of the force polyhedron in this case is zero, since the resultant face is coplanar with the faces of the applied loads (Figure 7(g)).

Constructing a funicular polyhedron. Once the closed force polyhedron is constructed, it should be decomposed similar to the 2D example and the concurrent case of Figure 7(h). The funicular polyhedron can therefore be constructed similar to the concurrent case of Figure 7(i)and (j). Note that the edges of the funicular polyhedron intersect at the line of action of the resultant.

Funicular form finding for a given boundary condition and applied loads

In 2D graphic statics, the form diagram is constrained to the support locations and passes through the line of action of the applied loads (Figure 8(a)). The force diagram for the specified boundary conditions, on the other hand, is constrained to a line l which is perpendicular to the line connecting two support locations referred to as the *closing string* (Figure 8(b)). Moving the point p on the line l results in various funicular solutions that are constrained to the given boundary conditions.

In 2D graphic statics, finding these constrained form and force diagrams for the specified boundary conditions includes the following geometric procedures:[5]

- Constructing a closed force polygon for the given loading condition;
- Finding the location of the line l through trial funicular construction;
- Decomposing the force polygon to a point on the line l;
- Constructing the funicular form using the direction of the edges of the force polygon.

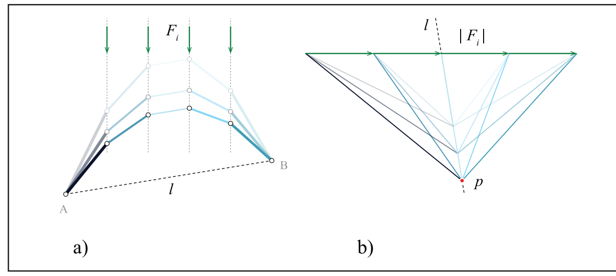

Figure 8. (a) Two-dimensional funicular form constrained to the line of action of the applied loads and the support locations and (b) the corresponding force polygon is constrained to a line *l* perpendicular to the line connecting the support locations.

The following examples describe the geometric process of finding funicular form for a determinate boundary condition consisting of three support locations in 3D. The first example specifically chooses each support location coplanar with an applied load and the resultant, whereas the other example describes the form-finding process for the support locations that are placed arbitrarily around the applied loads in the 3D space.

Support locations coplanar with an applied load and the resultant force

In this example, the location of each support has been constrained to a plane passing through at least one applied force and the line of action of the resultant (Figure 9(a)). The force polyhedron of this example is identical to the force polyhedron of Figure 5(e), where the line of action of the resultant is found using funicular construction explained in section "Concurrent forces."

In 2D, the trial force polygon is used to construct the trial funicular polygon and to define the location of the line l. Similar technique can be used to find the line l in 3D by constructing the trial funicular polyhedron. The force polyhedron of Figure 9(b) can be decomposed to an arbitrary point p_{trial} (Figure 9(d)); the result is called the *trial* force polyhedron which is the equivalent of the trial force polygon in 2D graphic statics.

The funicular polyhedron can then be constructed through the following steps: pick a point A_{trial} on a line parallel to the resultant and passing through the support A; from A_{trial}, draw a line perpendicular to face 1 intersecting the line of action of the force F_k; continue the drawing process until the trial funicular polyhedron is completed (Figure 9(e) and (f)). This trial funicular polyhedron passes through the line of action of the applied forces and is constrained to the parallel lines passing through the supports.

The 3D equivalent of the "closing string" in 2D graphic statics is a line connecting the support locations A_{trial},

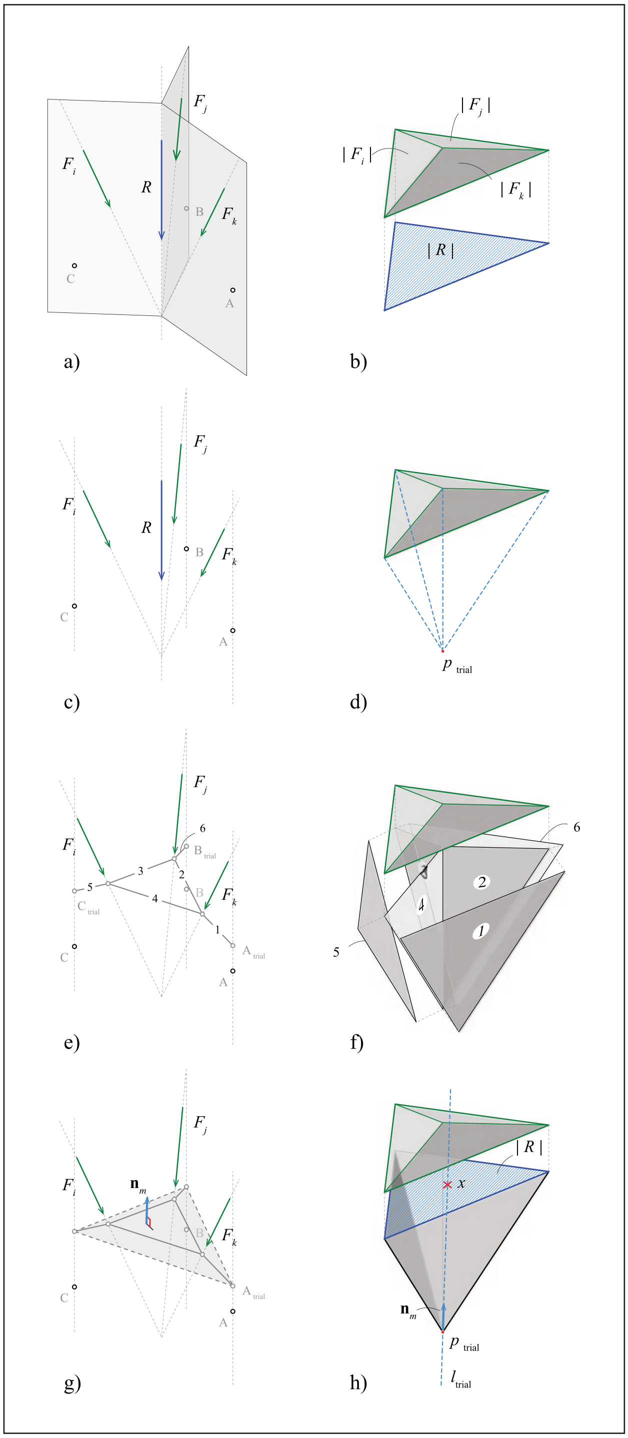

Figure 9. Construction of a trial funicular polyhedron for a concurrent set of loads and three support locations coplanar with the resultant and the applied loads: (a) the support locations and their planes, (b) the reciprocal force polyhedron, (c) the line of action of the resultant, (d) decomposition of the force polyhedron, (e and f) funicular polyhedral construction, (g) drawing the closing string connecting the support locations, and (h) drawing line l from point p_{trial} and its intersection with the resultant face x.

B_{trial}, and C_{trial}. This line establishes a closed polygon called the *closing plane* with a normal n_m (Figure 9(g)).[14] In fact, the trial force polyhedron is constrained to a line l_{trial} parallel to the normal n_m drawn from the trial point p_{trial}. This line intersects the resultant face at a point x which remains fixed for any arbitrarily chosen point p_{trial}. This property that has exact equivalent in 2D graphic statics hints to the fact that any other force polyhedron constructed by an arbitrary p_{trial} is therefore constrained to a line that certainly passes through the point x.[5,14]

In fact, there exists a force polyhedron constrained to a line l that is perpendicular to the closing plane connecting the points A, B, and C and passes through the point x. Therefore, the line l can be drawn from the point x parallel to n_p of the closing plane (Figure 10(a) and (b)). Picking a point p on the line l can result in a force polyhedron for a funicular solution that passes through the applied loads and is constrained to the support locations A, B, and C (Figure 10(c) and (d)). Moving the point p on the line l results in various (compression or tension) funicular forms and their corresponding force distribution for the given boundary conditions (Figure 10(c) and (f)).

Support locations arbitrarily chosen around the applied loads

Consider a case where the support locations are arbitrarily chosen around the applied forces (Figure 11(a)). Note that to find a spatial funicular form for the given support conditions, the loads must be applied within the area bounded by the support locations. According to Akbarzadeh et al.,[13] the edges of the form diagram in 3D reciprocal polyhedra share planar open/closed faces. Therefore, the edges of the funicular solution must be on the planes passing through the support locations and the applied loads. These planes are named with their normal direction as $\mathbf{n}_m$, $\mathbf{n}_p$, $\mathbf{n}_q$, $\mathbf{n}_r$, $\mathbf{n}_s$, and $\mathbf{n}_t$ (Figure 11(c)).

The force polyhedron for this example is not unique (geometrically indeterminate). To construct a force distribution for the given boundary conditions, we can start from the closed force polyhedron of Figure 11(b). Draw lines perpendicular to the planes passing through the support locations from the vertices v'_r, v'_t, and v'_s of the force polyhedron. These lines intersect at the vertices v'_j, v'_k, and v'_l and therefore construct faces corresponding to the applied loads and the given boundary conditions (Figure 11(d) and (f)). To find the line l, we can use the techniques described in section "Support locations coplanar with an applied load and the resultant force" (Figure 11(g) and (h)). Once the line l is defined, the faces of the force polyhedron can be decomposed into a point on the line l (Figure 11(j)), and a funicular form can be constructed that is constrained to the boundary conditions.

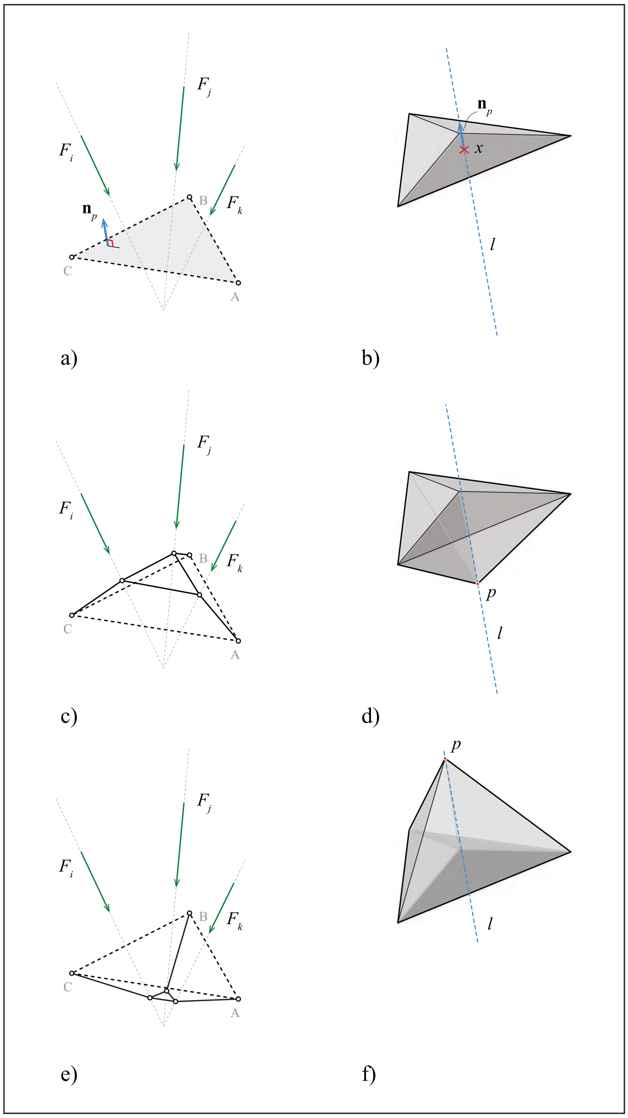

Figure 10. Construction of the constrained funicular form: (a) constructing the closing plane and finding its normal direction, (b) drawing line l parallel to the normal of the closing plane, (c and d) constructing a compression-only funicular form by decomposing the force polyhedron to the point p on l, and (e and f) moving the point p on l results in tension-only funicular form and its corresponding force polyhedron.

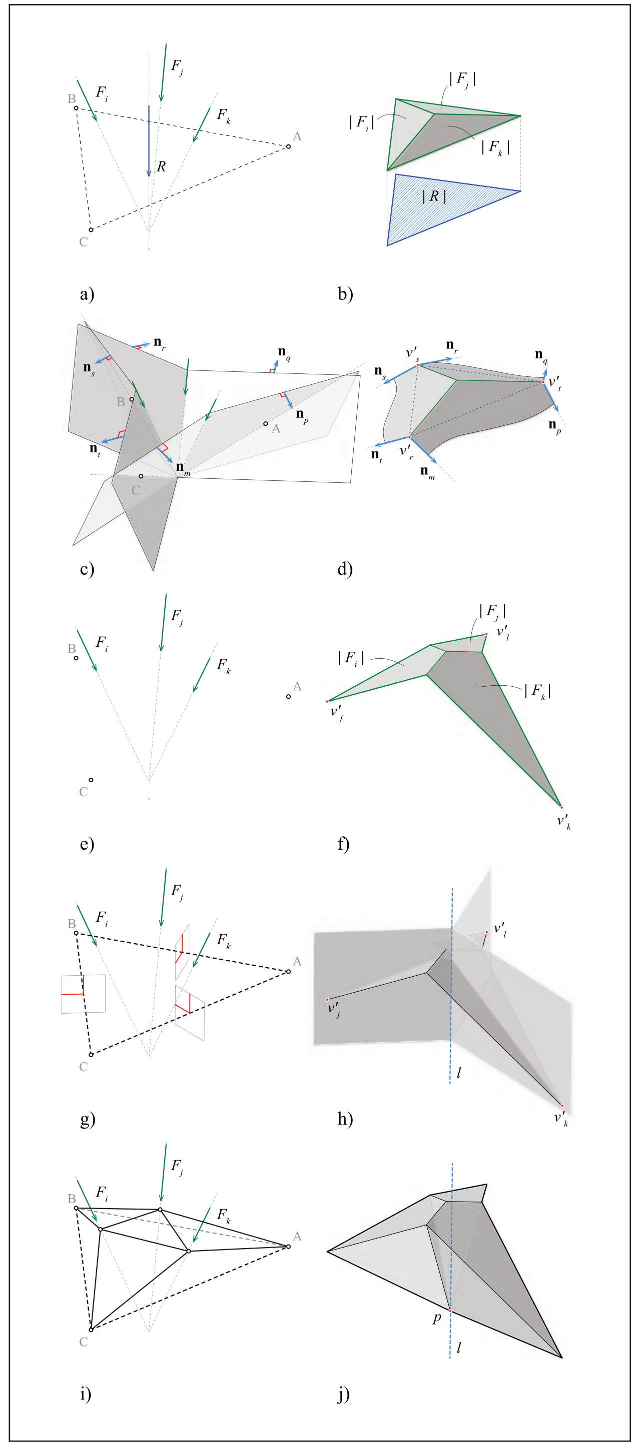

Figure 11. Funicular form finding for the support locations freely chosen around the applied loads: (a) the concurrent set of loads and the given support locations, (b) closed force polyhedron for the applied loads, (c) planes passing through the applied loads and the support locations, (d) constructing the force polyhedron for the given boundary condition, (e and f) the faces of the force polyhedron corresponding to the applied forces surrounded by the support locations, (g and h) process of finding the line constraint, and (i and j) the funicular form and force polyhedron for the given boundary conditions.

Discussion and future work

This article investigated how polyhedral reciprocal form and force diagrams can be used to develop a 3D graphical method that provides step-by-step procedures for solving fully spatial structural design problems, similar to the graphic statics procedures for 2D systems of forces. As a first step, it provided procedures for establishing global equilibrium for given boundary conditions and for constructing funicular solutions for parallel loads. The importance of the "closing plane" and the "point X" was also discussed in this context.

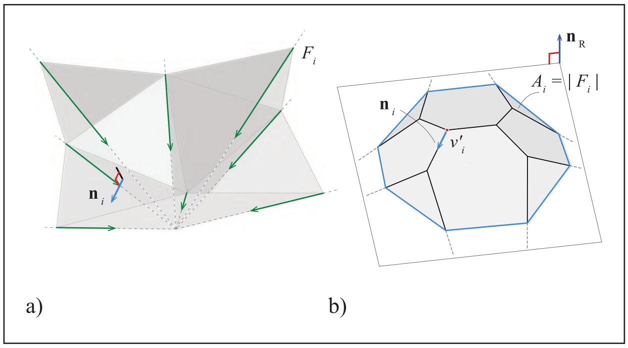

Figure 12. (a) Multiple concurrent applied loads and their shared faces and (b) the reciprocal, closed force polyhedron.

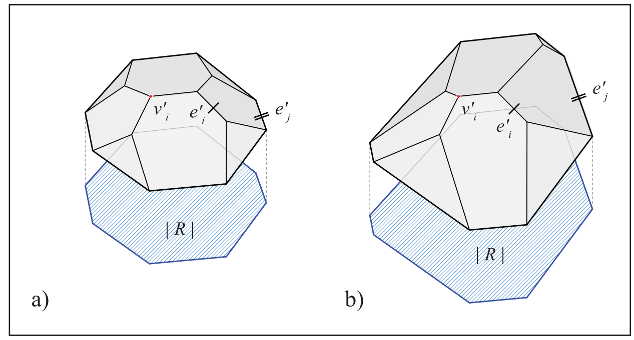

Figure 13. (a) A closed force polyhedron as a starting polyhedron constructed using the geometric procedures presented in this article and (b) another force polyhedron with different edge lengths and face areas highlighting the geometric degrees of freedom of the starting force polyhedron.

It is important to note that a 3D structural design method based on polyhedral form and force diagrams has the inherent limitation that it can only address structural problems that can be represented by these kinds of diagrams. Maxwell[3] provides a mathematical description of this limitation.

In addition, the provided procedure for constructing funicular solutions for given boundary conditions is currently limited to simple configurations of concurrent and parallel loads. In the following paragraphs, we discuss some of the difficulties of dealing with more complex loading conditions and with non-concurrent loads.

On the geometric degrees of freedom of the force polyhedron

This article mainly describes the geometric procedures for very simple parallel and concurrent loading cases. These procedures can be easily applied for multiple applied forces and their shared faces. Figure 12(a) illustrates multiple concurrent applied loads including their shared adjacent faces and a reciprocal force polyhedron constructed using the methods presented in this article (Figure 12(b)).

The force polyhedron for this case is not unique; it means that the edge lengths of the faces of the force polyhedron can be changed without changing the direction of the edges/faces. This property is due to the fact that the force polyhedron is geometrically indeterminate or it has geometric degrees of freedom.

Note that a change in the length of an edge alters the area of its adjacent faces. Since the area of each face corresponds to the magnitude of an applied force in the system, the change in the area of each face affects the areas of the adjacent faces and therefore the magnitude of the applied forces in the system (Figure 13(a) and (b)). However, we can use a meter to ensure that the direction of the resultant of all these forces stays constant; the closing face of the force polyhedron is perpendicular to the resultant force in the given system and is calculated by adding the lengths of the applied forces. Therefore, any change in the edge lengths and areas of faces of the force polyhedron does not change the direction of the resultant face (Figure 13(a) and (b)).

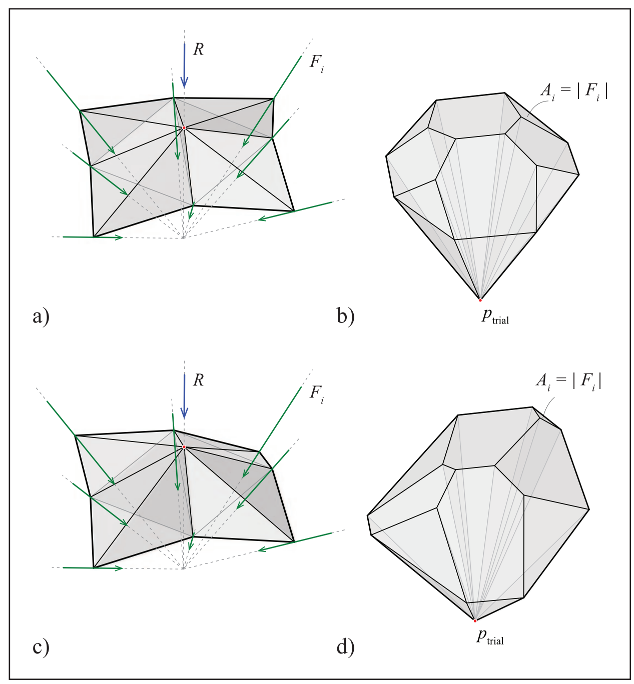

Figure 14. (a) A closed funicular polyhedron for the starting force polyhedron, (b) a decomposed force polyhedron, (c) a funicular polyhedron for the modified force polyhedron, and (d) the force polyhedron with the different edge lengths.

We can use both force polyhedrons to find the global equilibrium of the given force system by constructing closed funicular polyhedron (Figure 14(a)–(d)). Figure 14(a)–(d) shows that regardless of the change in the areas of the force polyhedron, they both result in a closed funicular polyhedron for the given loading condition. Moreover, both funicular polyhedrons specify the same location for the line of action of the resultant force. It is worth mentioning that the geometric degrees of freedom of the force polyhedron allow redistribution of the forces in the nodal equilibrium that can be addressed in future research.

Non-concurrent forces

This article provided the procedures to construct global equilibrium only for parallel and concurrent loading cases. These forces can be replaced by a single resultant force, and therefore, their force polyhedron can be closed with a single resultant face. The non-concurrent loading conditions, on the contrary, cannot be replaced with a single resultant face, and therefore, the process for constructing their reciprocal, closed force polyhedron requires further steps which is beyond the scope of this article and will be addressed in future research and publications.

Moreover, the use of 3D reciprocal diagrams in funicular form finding is not limited to compression/tension-only systems and can be used for design and analysis of complex structural systems including compression and tensile forces that will also be addressed in future research and its relevant publications.

Nevertheless, the geometric procedures described in this article provide sufficient means to generate complex funicular solutions, especially in combination with the polyhedral subdivision techniques that were addressed in another publication by Akbarzadeh et al.[15]

Given the fact that the methods of 2D graphic statics are used and researched in multiple aspects after almost 200 years from its original development, the 3D equivalent of these methods promises years of development and research in the field of 3D funicular form finding and geometric optimization.

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