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Effect of Subdivision of Force Diagrams on the Local Buckling, Load-Path and Material Use of Founded Forms

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Abstract

This paper investigates the relationship between the topology of a structure, load-path values and material efficiency for given boundary conditions in structural form finding using 3D Graphic Statics (3DGS) methods. Subdividing the force polyhedron is a technique in graphic statics that allows generating topologically-different structural forms for a given boundary condition. This method is used to deal with buckling problems in long members by substituting them with multiple members with shorter lengths. However, subdivision methods result in more members and nodes in the structure and this adds to the construction costs and material use. This paper investigates the effect of subdivision techniques on the change in the load-path values and local buckling load for various developed funicular polyhedral systems and the volume of the construction material. Multiple subdivision algorithms are developed to generate series of bar-node compression-only spatial structural systems for a given boundary condition, and relevant algorithms are designed to calculate the volume, load path and maximum local buckling force. The results of 41 different specimens show that by applying subdivision on global force diagram, generally the maximum local buckling force would increase, as well as load path and volume. However, the slope of increase in local buckling force is higher. Furthermore, subdividing the applied forces as well as internal forces causes a better local buckling force than the subdivision of interior geometry.

Keywords: Force Polyhedral Subdivision, Local Buckling, Load Path, Material Use, 3D Graphic Statics

1. Introduction

Graphic statics has been used to design, analyze or optimize possible structural solutions for many years. This method allows designer to visually explore both form and force simultaneously, using reciprocal form and force diagrams, have control on internal force distribution, and maintain inside optimum solutions from early stages of design [1]. Due to the advancements of computational tools and the discoveries in 3DGS, now we have the potential of using this method to explore fully three-dimensional structures [2] and design non-conventional innovative geometries which are in compression or tension only, under specific loading scenario [3-5].

1.1 Problem Statement

Subdivision of the global force polyhedron is a technique to develop different topologies and geometries [6], it keeps the force diagram convex and adds to structural capacity by substituting the long members with multiple nodes and shorter members [7, 8]. However, increasing the number of nodes, generally also adds to the complexity, material use and construction cost. There are infinite ways of subdividing the global force diagram. The increase or decrease of each of these values (ultimate buckling load, volume, etc.) varies from one subdivision to another and there is no exploration on the effect of different subdivisions on each of them.

1.2 Objective

This research is a step toward improving structural performance using subdivision. It investigates the effect of subdivision on local buckling force, volume, load path, and number of nodes and members.

2. Methodology

In this research a series of bar node models with specific features has been made based on 3DGS principles using different subdivision algorithms as well as algorithms for volume distribution in 3D space to observe and compare the change in local buckling force, volume and load path, by increasing the number of nodes and members in a unique loading scenario and a consistent overall proportion for all the specimens.

2.1 Boundaries and Constraints

The initial global force diagram is a diamond with 12 faces which results in 6 applied forces and 6 supports. The resultant force of the applied forces and the supports are on the same axis (Fig. 1).

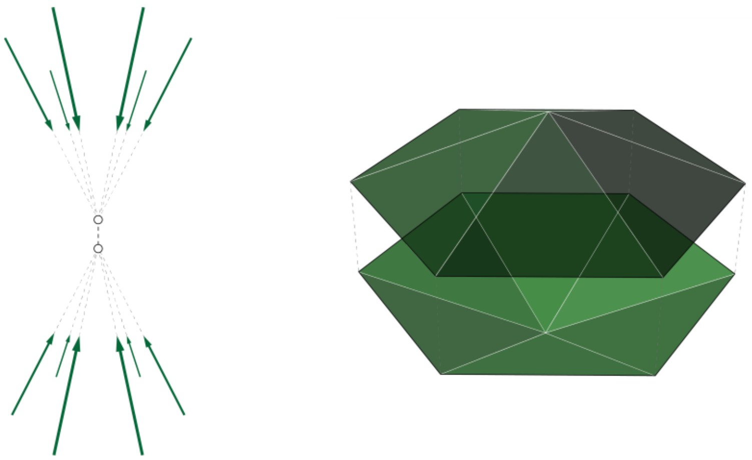

Figure 1: Initial form and force diagrams

Fabrication feasibility for later exploration and comparability are important factors in the process of generating these models. For this we must introduce some criteria: The proportion of all specimens are 152 d ×304 h mm

1. To maintaining the direction and magnitude of applied forces in all models, the summation of area of exterior faces and the angle of those faces are always constant
2. The diameter of members is between 2 to 5 mm
3. The maximum length of a member is 100 mm
4. The minimum length of a member is 1 mm
5. The maximum deviation for a member from the normal of its corresponding surface in the force diagram is 4º

2.2 Subdivisions

In order to study and compare the effect of different subdivisions, we have classified them into two types. The first type refers to subdivisions that hold the existing members and replace the nodes with new members and nodes. The second type replaces both existing members and nodes with series of new members and nodes.

It should be noted that depending on whether the external faces of the force diagram are subdivided or not, applied forces might be replaced with multiple smaller forces as well. Still, the total magnitude and the angle of those new forces are always the same and equal to the initial applied force. Figure 2 shows 8 different subdivision rules used in this paper.

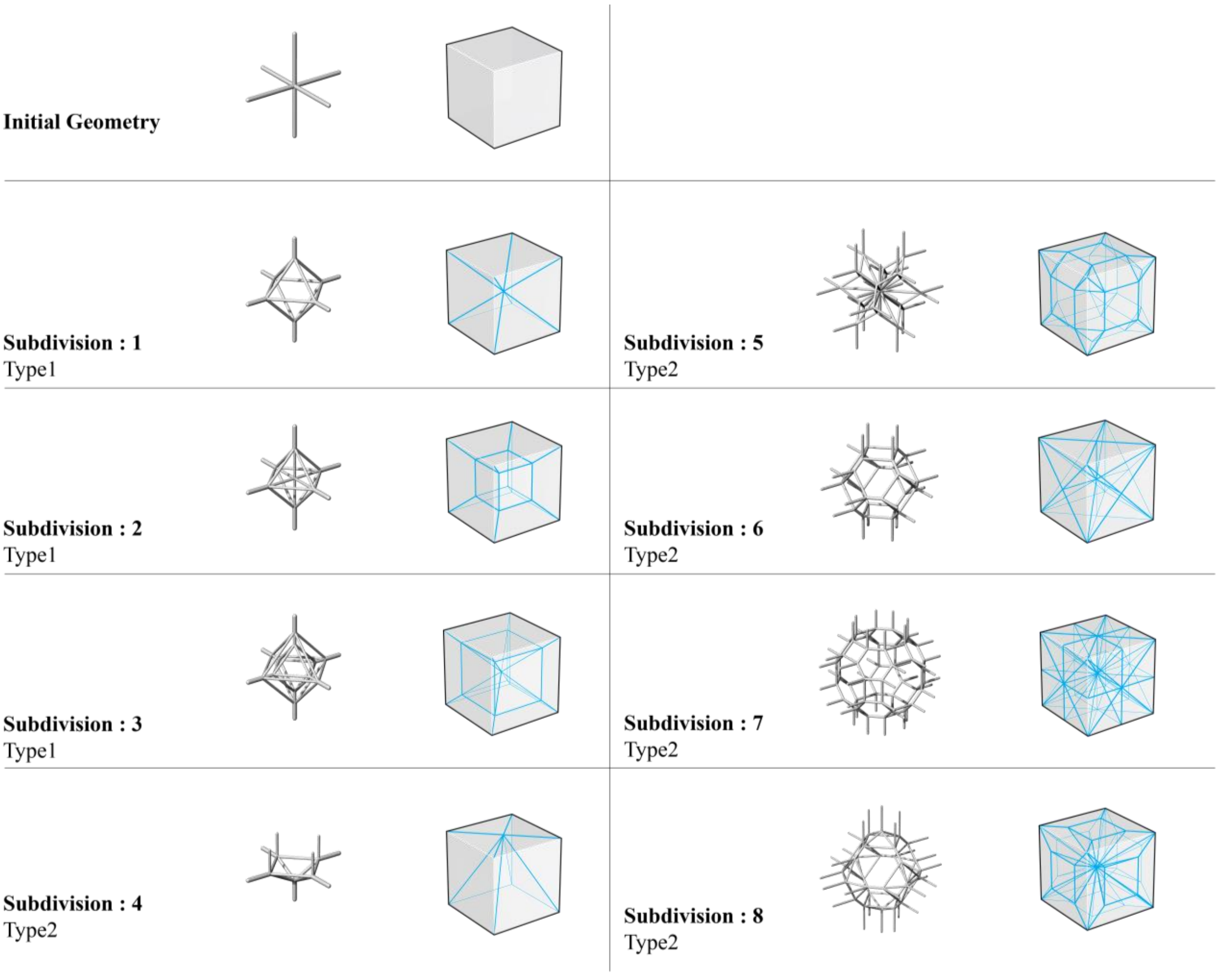

Figure 2: Developed subdivisions rules for later use on the force diagram

2.3 Volume

To calculate the volume, a radius is assigned to each member based on the ratio of magnitude of the member internal force to all members in all the other specimens. Then a sphere with the radius equal to maximum radius of its receiving members is assigned to each node. Each member is represented by a pipe between two spheres. Therefore, the length of members is shortened based on the radius of spheres at their both ends (Fig. 3).

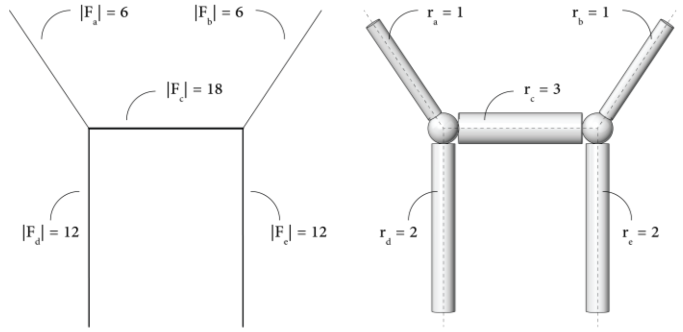

Figure 3: Generating volume

2.4 Specimens

The early explorations on application of these subdivisions on a node resulted in a dramatic increase in geometric complexity, number of nodes and members by implementing only one or two subdivisions on the specimen. As a result, most of the modules generated using aforementioned subdivisions did not pass the proposed criteria. So, to have more flexibility two different initial geometries are considered: a node and a member. Between the 112 unique produced force diagrams, 41 resulted in specimens that meet the criteria (Fig. 4). For a better comparison they are divided into 3 groups:

-specimens that resulted in no applied force subdivision

-specimens where the applied forces were subdivided at every subdivision;

-specimens where the applied forces were subdivided either in the first or in the second step of the process.

2.5 Load Path

It has been shown in 2D graphic statics that in a given structure, minimizing the load path results in minimizing the volume and generally finding the optimum solution [7,8]. However, in this paper the structural performance is studied for different topologies, so the minimum load path is not necessary the best performance and the variant with the smallest volume. This is why it is necessary to study and compare the change in load path versus volume and structural capacity as a result of applying subdivision. The load path is calculated for each specimen using equation 1:

$$\sum_{i=1}^{n}(|F_i| \times L_i) \quad (1)$$

where |F| is the magnitude of internal force in each member (or the area of its corresponding surface in the force diagram), and L is the length of that member.

2.6 Maximum Buckling Force

After assigning the radius to each member, buckling stress is calculated for all the members using Euler Formula (eq.2):

$$\sigma_{cr} = \frac{\pi^2 \times E}{\left(K \times \frac{L}{r}\right)^2} \quad (2)$$

where σ_{cr} is the critical stress, E is the modulus of elasticity (steel has been considered as the main material with E equal to 200GPa), K is the unsupported length of member which is equal to 1 for a member with simply supported ends, and L is the length of the member.

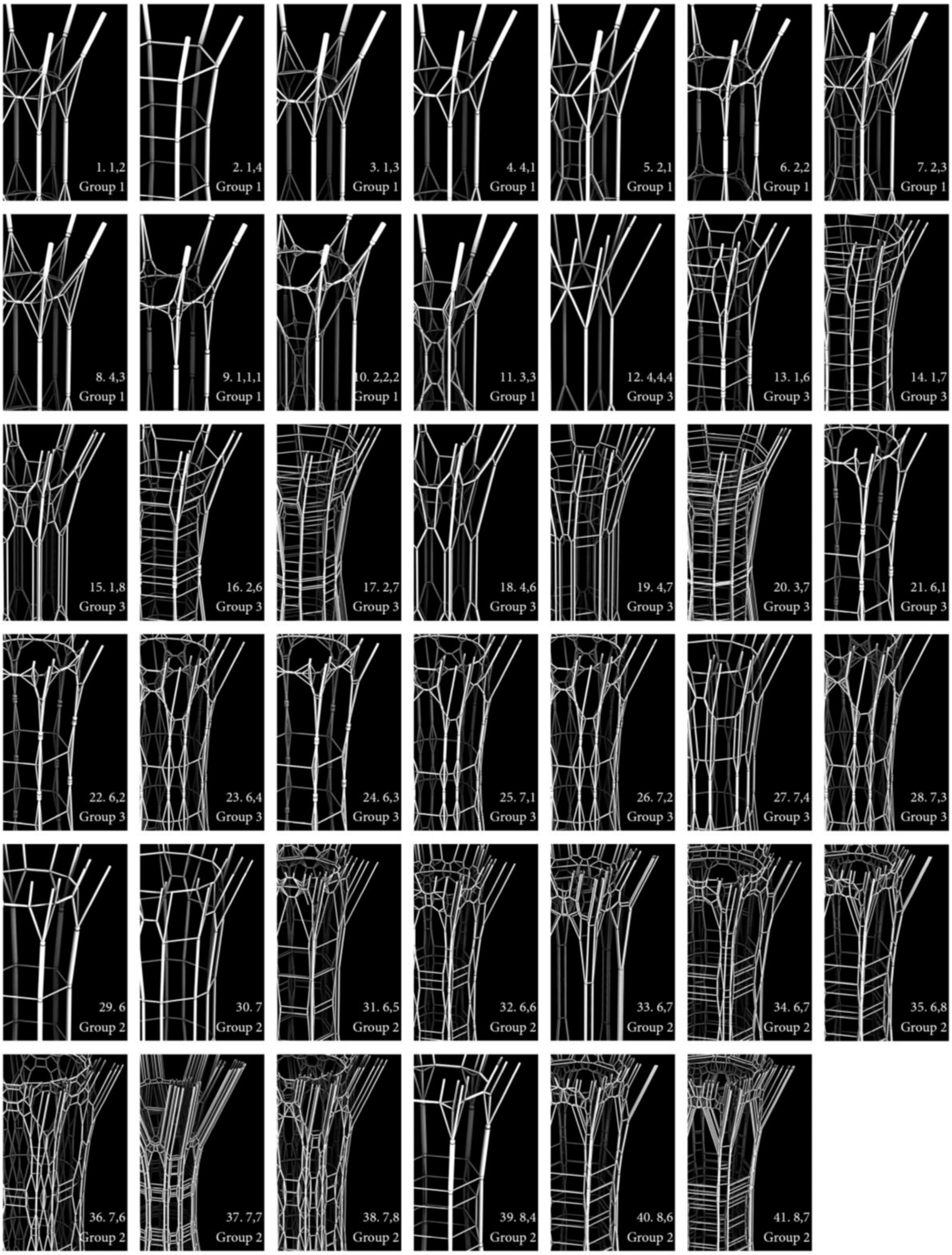

Figure 4: 41 Specimens that meet the criteria. For example, "subdivision: 1,8" means first subdivision 1 is applied and then subdivision 8 is applied on the result of the previous subdivision.

Buckling force can be simply found by multiplying the stress by the area of each member. The member with the minimum internal buckling force, defines the ultimate buckling load for local failure. Hence, to calculate the ultimate buckling load, the magnitude of applied load should be found based on the minimum internal buckling force in the system. The ratio of magnitude of the force between each two members is equal to the ratio of areas of their corresponding surfaces in the force diagram. Eventually, the summation of magnitude of applied loads is equal to:

$$\sum_i^n |F_b| \times \frac{A_i}{A_b} \tag{3}$$

where F_b is the minimum buckling force, A_i is the area of corresponding surface of applied force i, A_b is the area the corresponding surface of the member with the minimum buckling force.

3. Results

The following charts are the results of the study on 41 specimens generated by implementing 8 different subdivisions, or a combination of them, in a single global force diagram. As mentioned before, specimens are divided into 3 groups, therefore, the effect of subdividing the exterior and interior surfaces of the force diagram versus subdividing only the internal geometry can be compared.

By comparing these charts, we can see that although the complexity of the whole system and the number of nodes and members increases significantly by subdividing the applied loads (groups 2,3), this type of subdivisions always results in a better ultimate buckling load for local failure. In some cases, improved results of local buckling performance are observed with similar number of nodes and members. For instance, specimen 28 with 1764 members has 968.8 N ultimate local buckling force and specimen 33 with 1836 members has 2245N ultimate local buckling force. Which means the ratio of number of members is 1 to 1.04 but the buckling force is 2.3 times higher. The results also show that by keeping the applied forces and subdividing the internal geometry (group 1), ultimate local buckling does not increase.

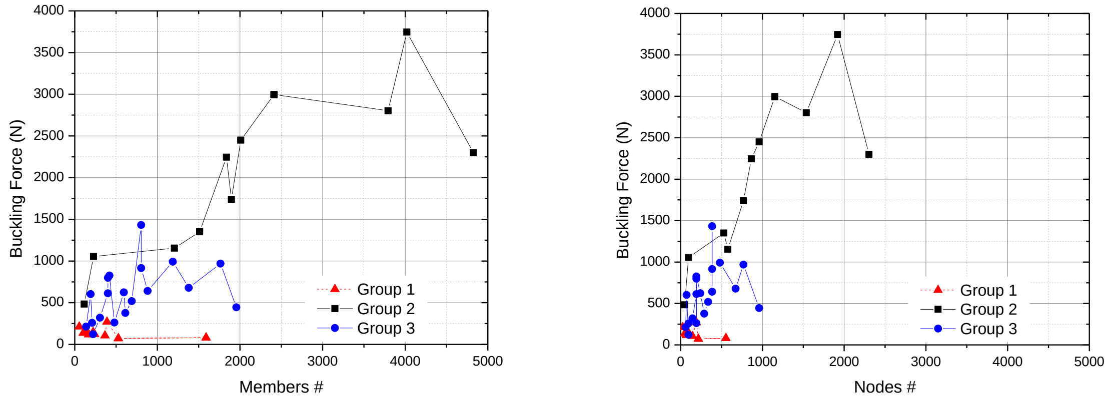

Figure 5: a) ultimate buckling force for local failure versus the number of members, b) ultimate buckling force for local failure versus the number of nodes

By subdividing the force diagram and adding more nodes and members, the volume would increase, however, the ratio of increase in maximum buckling force is usually more than the increase in volume. As it is shown in the charts, the ratio between maximum and minimum volume is around 2.3 but the ratio between buckling force of the same specimens is 3.6. By subdivision and adding to the number of nodes and members load path would increase (chart 2 and 3). However similar to volume, the increase in buckling load is more than the increase in the load path.

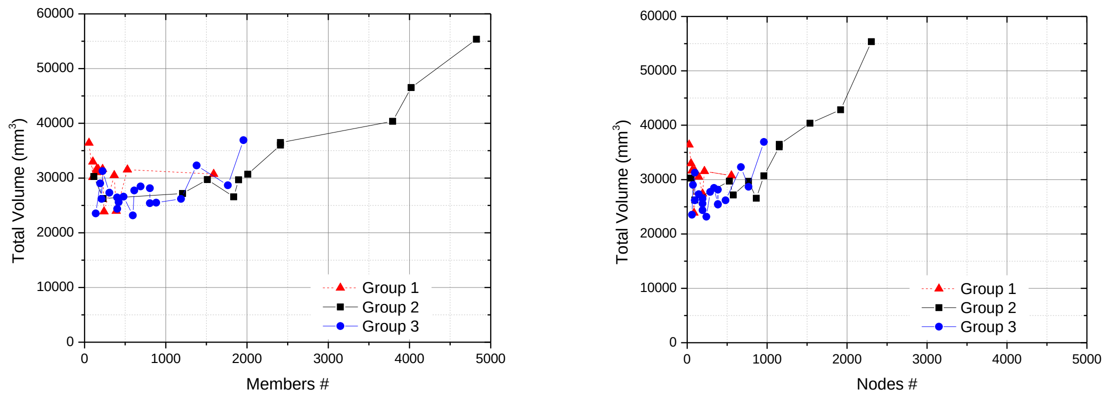

Figure 6: a) volume versus the number of members, b) volume versus the number of nodes

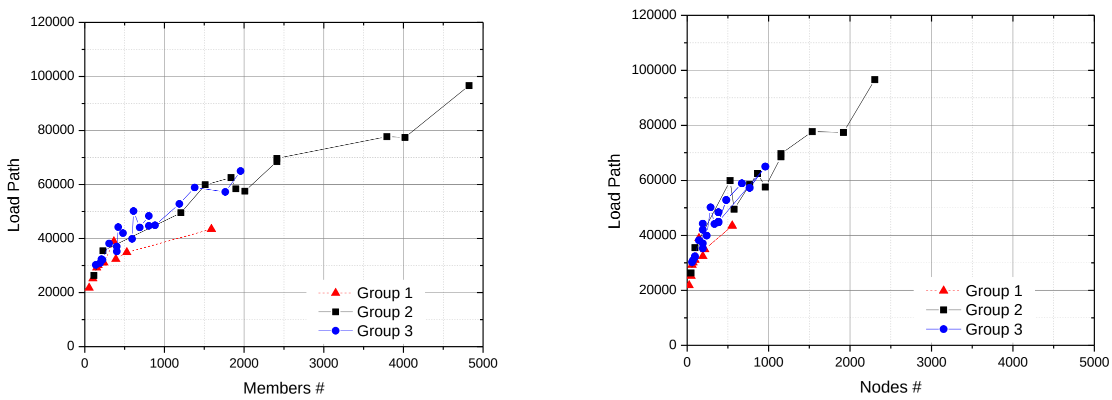

Figure 7 a) load path versus the number of members, b) load path versus the number of nodes

4. Eigenvalue buckling analysis

Three of the specimens (30, 32 and 41, see Fig. 8) are selected for linear buckling analysis to compare with the local buckling load found in the previous section. Linear buckling analyses have been done using SAP2000. The final buckling loads are 3.2, 1.9, and 4.1kN, respectively. Although local buckling analysis showed that specimen 32 is stronger than specimen 30 due to more subdivision, its global buckling load is less. This mainly can be associated with the change in the buckling mode from flexural to torsional or flexural-torsional. The same buckling mode occur in the specimen 41, however, due to more subdivision the stiffness of structure increases which leads to higher buckling load as well.

5. Conclusion

The results of 41 different specimens show that by applying subdivision on the global force diagram, generally, the load path and volume would increase. However, the maximum local buckling force increases if both exterior and interior surfaces of the force diagram are subdivided. If so, the slope of the increase in local buckling force is higher as well. Furthermore, subdividing the applied forces, in every application of subdivision, causes a better local buckling force than the subdivision of interior geometry. For instance, with the same number of nodes and members, the local buckling force would be 2.3 times higher.

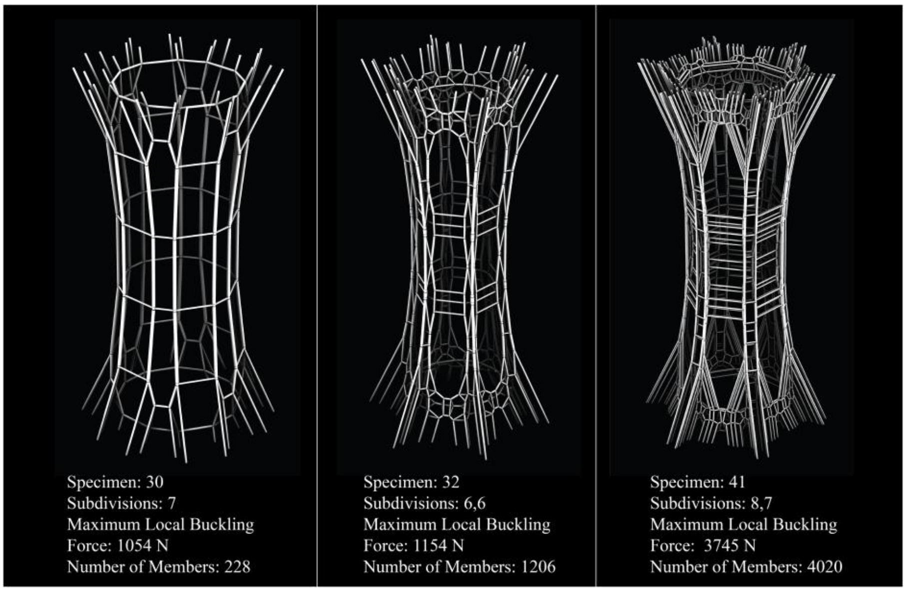

Figure 8. Three chosen specimens for linear buckling analysis.

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