The Design of an Ultra-Transparent Funicular Glass Structure

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ABSTRACT

This project presents novel research in structural design and analysis of an ultra–transparent pedestrain bridge made exclusively of glass sheets in a double layer, funicular, compression–only configuration. The funicular form of the bridge maximizes its structural performance and minimizes the use of materials and resources. The structural form of the project has been developed using 3D graphic statics (3DGS) that is a geometry-based structural design method allowing the extensive exploration of funicular structural solutions in three dimensions. Using the 3DGS method results in structural forms that are polyhedral geometries with planar faces. Therefore, not only does 3DGS find the efficient structural forms, but its planarity constraint facilitates the construction using flat sheet materials. The current structure of the bridge consists of three-dimensional polyhedral cells as hollow glass blocks with planar glass faces held together in compression by using transparent silicon-based substance (figure 1). The total span of the bridge is 10 m (32.81 ft) with a one-meter deck for pedestrian traffic. The asymmetric geometry of the bridge will significantly improve the behavior of the bridge under asymmetric and lateral loading conditions.

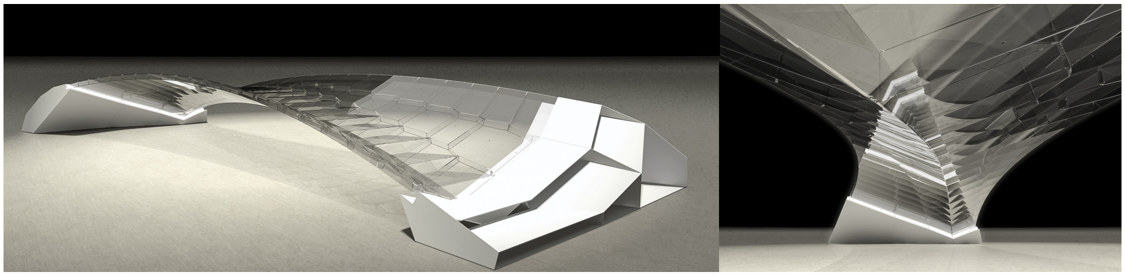

Figure 1: The materialization and early visualizations of the architectural design concept of the ultra-transparent glass pedestrian bridge (Copyright PSL 2018).

INTRODUCTION

Funicular structural forms maximize the structural performance and minimize the use of materials and resources. These systems carry the applied loads in the form of pure tensile/compressive axial forces such that the form/geometry of the structures matches its internal flow of forces. The Sagrada Familia by Antoni Gaudí is an excellent example of using such forms in design and engineering. Gaudí used tedious physical form-finding techniques to find such funicular forms for his breathtaking structures. However, many eminent engineers and designers such as Guastavino, Maillart, Eiffel, Nervi, Dieste relied on geometric methods of structural design, known as Graphical Statics (GS), to design their efficient structures. Graphical Statics (GS) methods represent a group of powerful and intuitive geometric techniques for form-finding and analysis that originated in the pre-digital era and continue to be used and developed even today (Maxwell, 1864; Culmann, 1864; Cremona, 1890; Wolfe, 1921; Block, 2009; Fivet and Zastavni, 2013; Akbarzadeh et al., 2014). In graphic statics, the equilibrium of each node of the structure can be represented by a closed polygon of force where the magnitude of the force in each member of the node is equivalent to the length of the corresponding edge in the force polygon. The force diagram of compression-only forms consists of closed, convex force polygons, therefore, finding compression-only funicular forms is quite intuitive using GS-based methods. for The structures designed by GS-based methods are among the best examples of innovative use of materials. Gustave Eiffel's tower, Maillart's Salginatobel bridge, Guastavino's ultra-thin load–bearing vaults with only two layers of tile brick are all designed using GS-based methods (Zastavni, 2008; Ochsendorf and Freeman, 2013; Block et al., 2018).

3D graphic statics

Despite its clear strength and advantages, traditional graphical statics were limited to 2D diagrams, and a designer could only design 2D abstraction of three-dimensional structures. In 2016, the methods of 2DGS were extended to 3D based on a 150-year-old proposition by Rankine (1864) in Philosophical Magazine (Akbarzadeh et al., 2015b,c,a; Akbarzadeh, 2016; McRobie, 2016). In 3D graphic statics (3DGS), the equilibrium of the external forces or a single node of an equilibrated structure is represented by a closed *polyhedron* or a polyhedral *cell* with planar faces. Each face of the force polyhedron is perpendicular to an edge in the form diagram, and the magnitude of the force in the corresponding edge is equal to the area of the face in the force polyhedron (Figure 2).

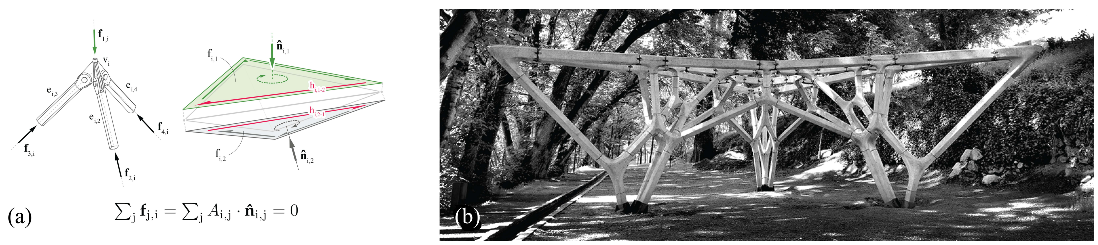

Figure 2: (a) A node v_i of a spatial structure in equilibrium; and the elements of the reciprocal cell c_i representing the equilibrium of the node v_i, with directions of normals and half-edges of the faces $f_{i,1}$ and $f_{i,2}$ shown; (b) the first built structure designed by using 3DGS methods (Akbarzadeh et al., 2017; Bolhassani et al., 2017) (Courtesy of PSL).

Compression-only glass structure: motivation and objectives

Using 3DGS methods in structural form finding results in lightweight structures with high-performance structural behavior (Bolhassani et al., 2017, 2018). Moreover, the resulting structural forms are polyhedral geometries with planar faces. Therefore, not only does 3DGS find efficient structural forms, but its planarity constraint facilitates the construction using flat sheet materials (Figure 3). To take advantage of the planar geometry of the reciprocal polyhedral diagrams in construction, this research will explore the structural efficiency and behavior of a lightweight, ultra-transparent glass pedestrian bridge constructed out of planar glass sheets in compression.

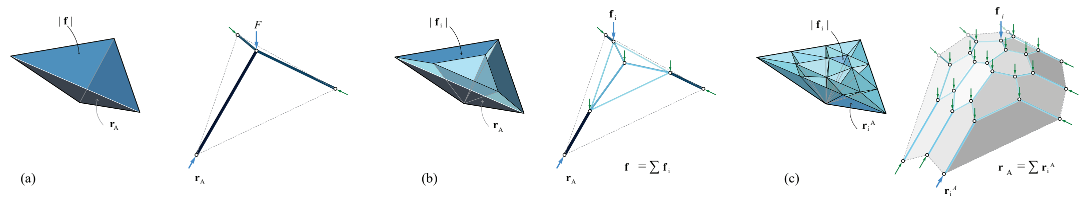

Figure 3: Changing the form configuration from a simplest compression-only form (a) to a shell with planar faces (b) and (c) using subdivision techniques (Akbarzadeh et al., 2015c).

RESEARCH METHODOLOGY

The research presented in this paper is an on-going investigation in the design and construction of a fully transparent glass bridge designed by 3D graphic statics. The methodology section of this paper will serve as an introduction to the current investigation of this project and will cover the research strategies including form finding, feasibility analysis, and proposed robotic fabrication and assembly methods of the project.

Form finding

To design the compression-only form of the bridge using 3DGS, we started by aggregating closed convex force polyhedron in two stacked layers as shown in Figure 4b (Akbarzadeh et al., 2015b). PolyFrame beta (Nejur and Akbarzadeh, 2018) plugin for Rhinoceros software (McNeel, 2014) was used to generate the form and the force diagram for this design. Using this plugin, a designer can extract the topology of a structural form which is the mathematical dual of the input force polyhedrons. Then the edges of the form must be aligned perpendicular to the faces of the input force diagram to establish the reciprocity between the two which means the equilibrium of the structural form 4a. In the form finding process the total applied force (the area of all the top faces of the force diagram shown in green) is considered to be $1kN$. As a result, the internal and the reaction forces at the supports can be found as a fraction of the applied force. The total self-weight of the bridge, calculated by considering the density of the construction materials, can then be multiplied by the total applied load and the reaction forces can be calculated based on the self-weight of the structure. As it is illustrated in the Figure 4, the magnitude of the internal forces at the bottom edges are much higher than the magnitude of the forces anywhere else. Note that, we intentionally designed the force diagram such that the form would have a big crease (fold) through the span of the bridge. This technique will increase the effective depth of the whole arched structure.

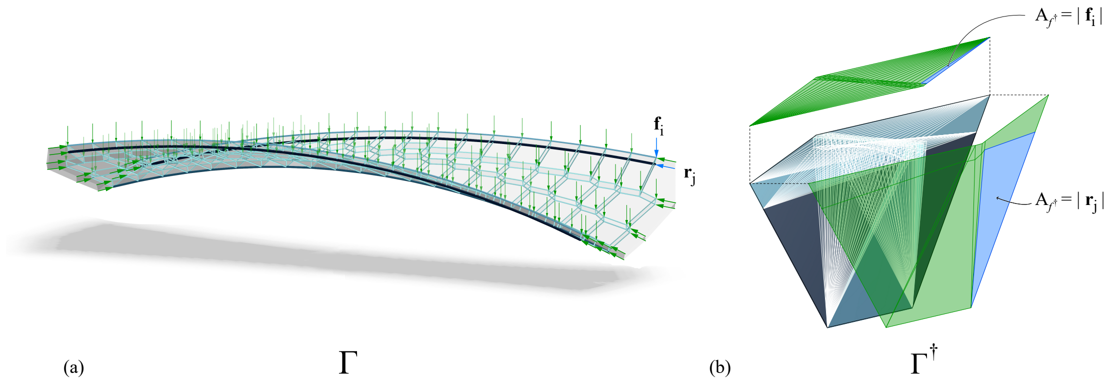

Figure 4: (a) The form (Γ) and its force ($\Gamma^\dagger$) diagram of the bridge developed by using 3D graphic statics and PolyFrame plugin (Akbarzadeh et al., 2015b; Nejur and Akbarzadeh, 2018) (Courtesy of PSL).

Design specifications

The ultimate design of the bridge is a structure spanning 10 m (32.81 ft) from two metal supports with a one-meter wide, glass, deck for pedestrian traffic. The asymmetric geometry of the bridge will significantly improve behavior under asymmetric and lateral loading conditions. The body of the bridge consists of hollow three-dimensional polyhedral cells as glass blocks with planar faces attached using a transparent structural acrylic adhesive bonding material (figure 5).

Structural/feasibility analysis

Preliminary analysis for live and dead loads

A preliminary linear FE analysis of the bridge was performed using continuum quadratic tetrahedral elements, type C3D10 (Figure 6). The effect of joints were not included in the study. Supports are modeled as pin connections and a 4 kN/m^2 distributed live load on the pathway has been applied. The results showed a maximum deflection of 0.87 mm (0.034 in) at mid span of the whole structure. Maximum tensile stress on the deck surface subjected to live load is 6.9 Mpa (1 ksi) which is less than the tensile strength of structural glass, see Figures 7 and 8.

Natural frequency of the structure

A linear perturbation analysis is performed to find the first four natural frequencies of the structure. The results of linear static analyses at this stage (modes

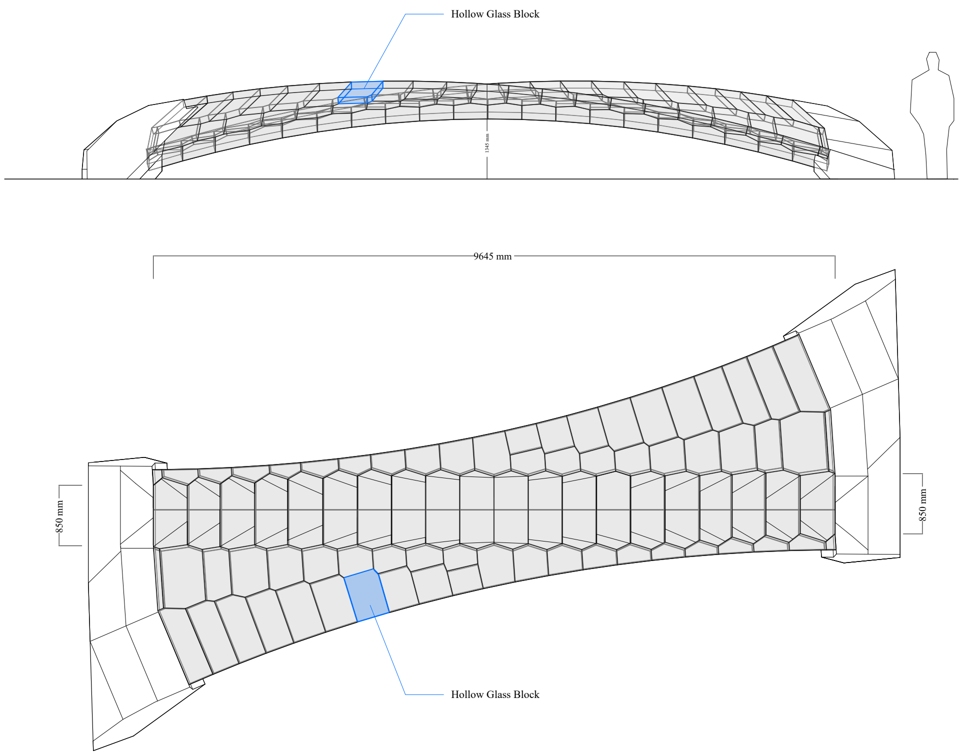

Figure 5: The plan and elevation of the glass bridge highlighting a Hollow Glass Block as a construction unit of the structure (Courtesy of PSL).

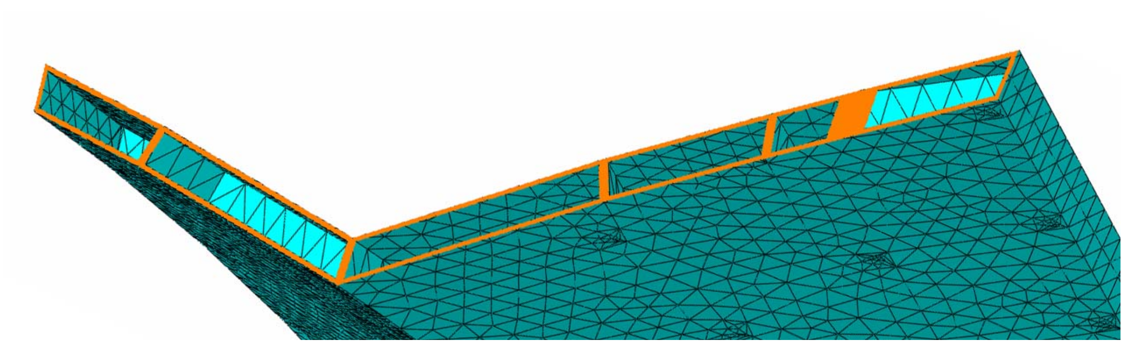

Figure 6: The tetrahedral mesh of the model used for FE analysis (Courtesy of PSL).

and frequencies) will be ultimately used as initial conditions in buckling eigenvalue analyses of the bridge. Results of this analysis are shown in Figure 9. The structure has shown a very high frequency in its first mode. This is likely due

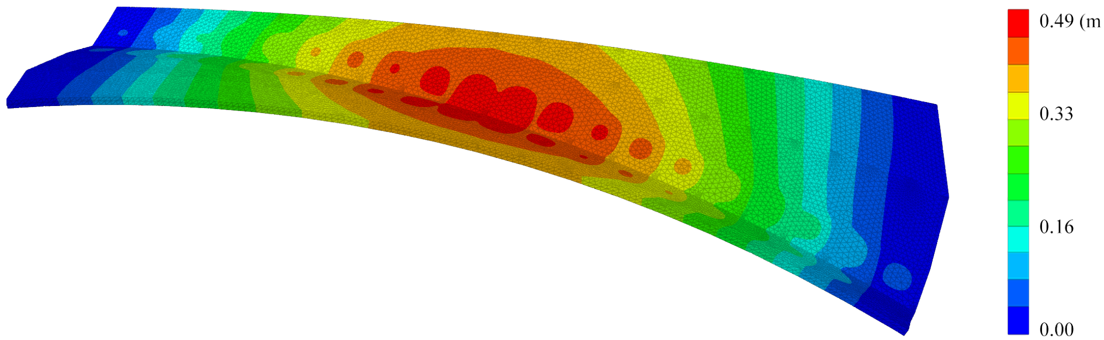

Figure 7: Numerical results of static analysis-deformation in mm (Courtesy of PSL).

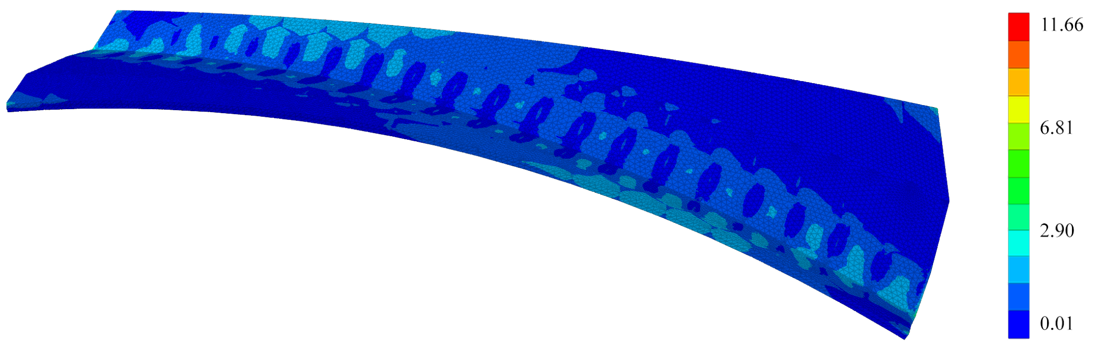

Figure 8: Numerical results of static analysis- Contour plot of distribution of Von Mises stress (Courtesy of PSL).

to the nature of the compression-only design: high stiffness and low mass. In sum, the FE analysis proved the feasibility of the construction of the project. However, further numerical/parametric modelings and physical load testings are needed.

Materialization and fabrication

Constructing the 10 m (32.81 ft) bridge with thin glass is proposed using a unique modular approach. That is, the 10 m (32.81 ft) bridge will be built out of a collection of three-dimensional hollow glass blocks (HGB). In this way, construction of the bridge is greatly simplified and reduced from assembling 462 individual glass plates to assembling 98 individual HGB. Furthermore, the modular construction approach has the advantage of improved quality control of the prefabricated HGBs, the opportunity for removal and replacement of damaged parts, and accelerated construction time.

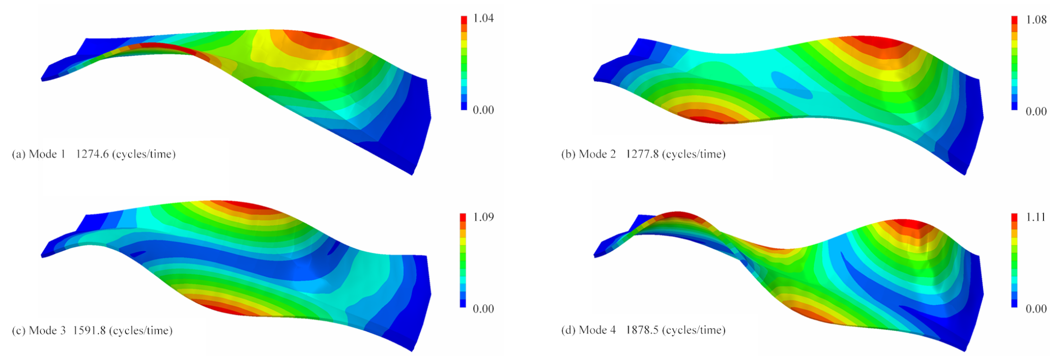

Figure 9: Multiple modes of the natural frequency of the structure (Courtesy of PSL).

CONCLUSION AND FUTURE RESEARCH

Our preliminary design, analysis, and results confirm the feasibility of the construction of an ultra-transparent glass pedestrian bridge. However, the proposed modular construction technique requires a clear understanding of the behavior of a single HGB when acted on by different load cases, as well as an understanding of the behavior regarding how the HGBs function collectively as a structural system. To develop these understandings, a two-stage research methodology is proposed where numerical modeling and experimental testing of a smaller bridge prototype will be performed. In stage-1, a typical HGB will be investigated. Critical in stage-1 is developing the connection details between the individual glass plates and understanding how these connections affect the behavior of the single unit under various loading scenarios. In stage-2, the connections between individual glass units and the assembly logic will be developed and used in construction and investigation of a small prototype. The objective of research stages-1 and -2 is developing a precise knowledge of the strength and stiffness characteristics of the proposed modular construction methodology.

The first logical step in further development of the research is to investigate the fabrication constraints and techniques in constructing a single HGB using flat sheets of 10 mm annealed glass. The structural properties of the glass, its thickness including the laminated layers, profile geometry and the type of adhesive substance to construct the block will be developed in collaboration with the team at the Technical University of Darmstadt, Seele and WARDjet companies. Figure 10 highlights some ideas in constructing a single HGB using various edge geometries. Investigating the structural performance of the HGB, includes both numerical modeling and experimental testing under a variety of different loading scenarios.

As mentioned earlier, the bridge consists of prefabricated parts mechanically en-

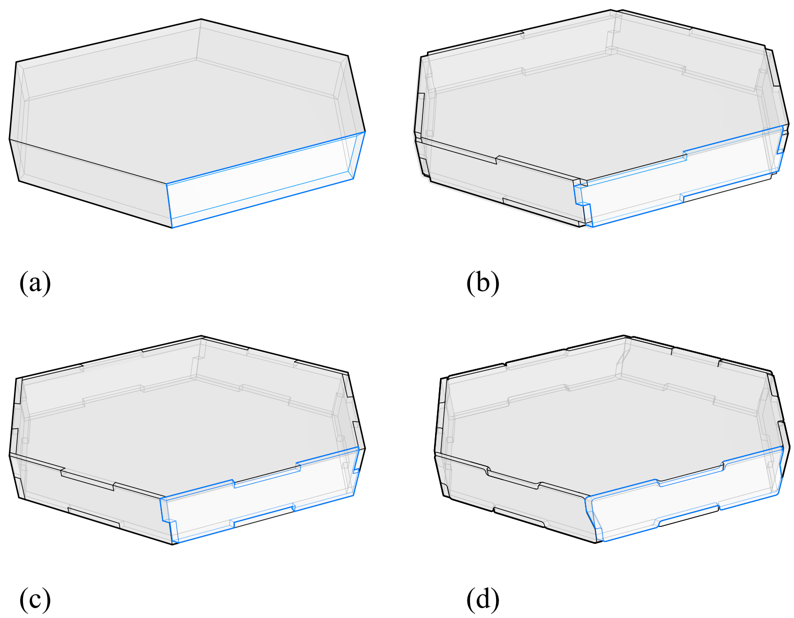

Figure 10: Each hollow glass block should be fabricated out of planar glass plates with specific edge geometry to construct a precise glass block for further structural performance.

gaged to complete the structural system. This will require developing a proper interlocking mechanism to efficiently transfer the axial and shear forces between the neighboring HGBs. The geometry of the interlock, the fabrication tolerance, and the assembly constraints due to the complexity of the geometry are key parameters to investigate. Moreover, a possible collision prohibition of the HGBs during the assembly and dis-assembly should be carefully addressed. Figure 11 shows two initial ideas to design the interlocking mechanisms for the assembly of the HGBs.
Investigating the structural performance of the aggregated bridge includes both numerical modeling and experimental testing under (a)symmetric loading scenario(s) that will be defined during the collaborative endeavour (Figure 12).

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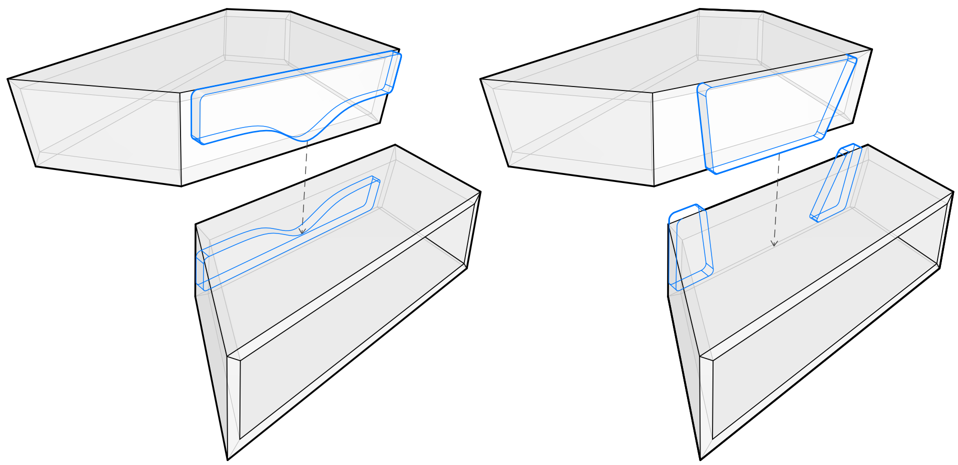

(a)

(b)

Figure 11: Two different interlocking ideas to engage/assemble the blocks in the bridge geometry.

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Figure 12: The 1:10 scale structural model of the bridge constructed using flat acrylic sheets (Courtesy of PSL).

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