Proceedings of the International Association for Shell and Spatial Structures (IASS)
Symposium 2015, Amsterdam
Future Visions
17 - 20 August 2015, Amsterdam, The Netherlands

Three-dimensional Compression Form Finding through Subdivision

Masoud AKBARZADEH*, Tom VAN MELE[a], Philippe BLOCK[a],

* ETH Zurich, Institute of Technology in Architecture, Block Research Group
Stefano-Franscini-Platz 5, HIL H 46.2, CH-8093 Zurich, Switzerland
amasoud@ethz.ch

[a] ETH Zurich, Institute of Technology in Architecture, Block Research Group

Abstract

This paper explains a geometric form-finding approach based on subdividing 3D global force diagrams to generate spatial compression-only forms. It is the three-dimensional extension of the two-dimensional methods presented in Akbarzadeh *et al.* [2]. The force diagram of a compression-only structural form consists of a (group of) closed, convex polyhedral cell(s) (Rankine, [9]; Akbarzadeh *et al.* [1, 3]). Similar to 2D, the force diagram consists of external and internal components, which respectively represent the global and local equilibrium of the spatial system of forces. This research introduces several subdivision schemes to subdivide the global force polyhedron of the 3D force diagram. This approach allows generating various novel typologies of compression-only, spatial structural form for given boundary conditions. The paper concludes by giving the approach practical relevance by relating the subdivision rules to buckling constraints of the members of the spatial structures being generated.

Keywords: force polyhedron subdivision, compression-only spatial forms, 3D graphic statics, 3D reciprocal form and force diagrams, form finding.

1. Introduction

In graphical methods for structural design, manipulating the geometry of the force diagram is quite effective to derive optimized and innovative structural forms. For instance, constraining specific edge lengths of the force diagram to a certain value results in a structural form with constant forces in the corresponding members (Allen and Zalewski [5]), optimizing the edge lengths of a force diagram in a truss can result in volume optimization of its structural components (Beghini *et al.* [6]), and, changing the geometry of the (horizontal) force diagram in the Thrust Network Approach can result in a variety of interesting design features in free-form shell structures (Block [7]).

As presented in Akbarzadeh *et al.* [2], subdividing the global force polygon in two-dimensional graphic statics is another technique to derive various funicular forms for given boundary conditions. In this approach, the "external" polygon, representing global equilibrium, is subdivided into smaller polygons using various subdivision schemes. The result is a variety of compression-only forms with different topological properties for the same boundary condition (Figure 1).

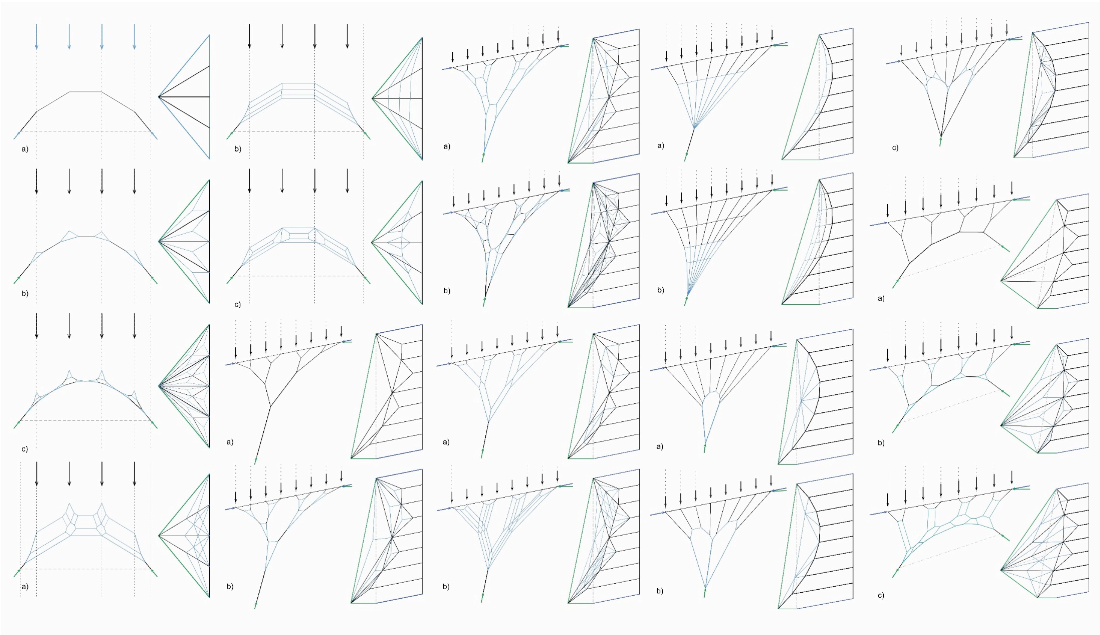

Figure 1: Two-dimensional form finding of compression-only structures using subdivision, from Akbarzadeh *et al.* [2].

In three-dimensional graphic statics, the force diagram of a compression-only structural form consists of a (group of) closed, convex polyhedral cell(s) (Rankine, [9]; Akbarzadeh *et al.* [1, 3]). Similar to 2D, the force diagram consists of external and internal components, which respectively represent the global and local (nodal) equilibrium of the spatial system of forces. Specifically, the areas of the faces of the force diagram represent the direction and the magnitude of the forces in the corresponding members in the form. The reciprocal relationship between the form and the forces in 3D, equivalent to two-dimensional graphic statics, suggests that the subdivision techniques used in 2D can also be used and developed for the spatial form and force diagrams.

This paper will explore the idea of subdividing the external/global force diagram in 3D as a design technique for form finding of spatial compression-only structural forms. Through a range of examples, it will furthermore highlight the possibilities this method offers to discover expressive and/or different funicular structures.

2. Methodology

This section briefly summarizes the concept of global and local (nodal) equilibrium in compression-only reciprocal form and force diagrams in 3D. Subsequently, it introduces two strategies for subdividing the global force polyhedron: subdividing only the face corresponding to the resultant force, and subdividing the internal space of the global force polyhedron without subdividing the external faces.

2.1. Global equilibrium / force polyhedron

For three-dimensional reciprocal form and force diagrams, the global equilibrium of a spatial compression-only form is represented by a convex polyhedron, which provides the magnitude and the direction of the reaction forces at the supports for a given resultant force (Akbarzadeh *et al.* [4]).

In a determinate system of forces, as the one depicted in Figure 2, the global force polyhedron is a tetrahedron. The faces of this tetrahedron correspond to the resultant of the applied loads, R, and the reaction forces at the supports, R_i, R_j, and R_k.

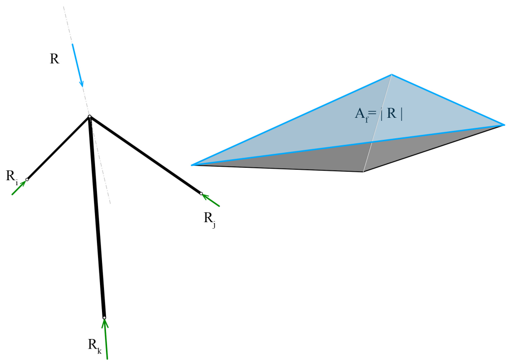

Figure 2: The form (left) and the tetrahedral force (right) diagram for a determinate system of forces in 3D.

2.2. Subdividing the resultant face of the global force polyhedron

The resultant face of the global force tetrahedron is the face that corresponds to the resultant of the applied loads. Any two-dimensional subdivision of the resultant face preserves the magnitude and the direction of the reaction forces at the supports. This property can be used as a design strategy to generate, through defined support points, interesting, compression-only support structures that carry a stiff, heavy plate (Lachauer and Block [8]).

Various two-dimensional methods of subdivision can be used to subdivide the polygon of the resultant face (Warren and Weimer [10]). Note that in this technique the edges of the face need to stay intact, since subdividing the edges would not preserve the boundary conditions of the supports.

As an example, consider the three-bar, statically determinate system in Figure 2 and its global force tetrahedron. The face corresponding to the resultant force can be subdivided into smaller polygons using different subdivision rules. Connecting the vertices of the newly created polygons to the apex of the tetrahedron results in an internal subdivision of the force tetrahedron. Doing this using different subdivision schemes and rules, various spatial funicular forms can be generated (Figure 3). For the different design examples, the magnitude of the resultant force and the reaction forces stay constant, but the location of the applied loads on the form can be changed.

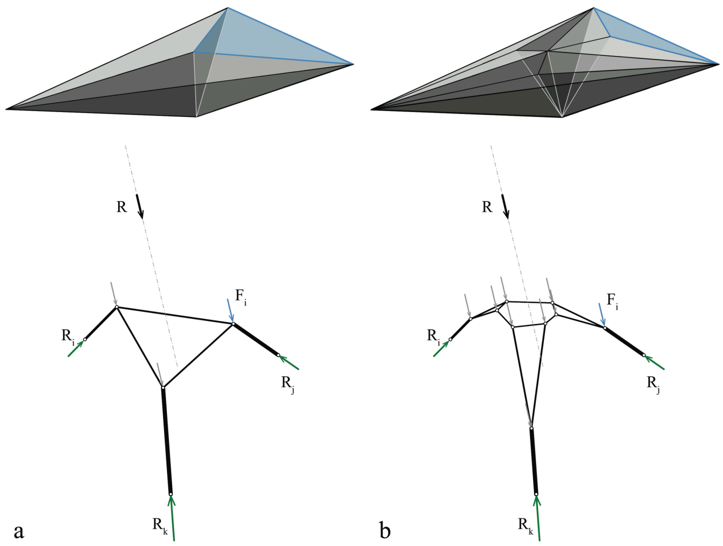

Figure 3: Multiple subdivision schemes that result from subdividing the resultant face of the global force tetrahedron of the simple system in Figure 2**.**

2.1. Subdividing the internal cell of the global force polyhedron

Subdividing the global force polyhedron without changing the external faces merely changes the internal distribution of the forces and not the location, magnitude or direction of the applied loads. The result is a new compression-only funicular form with the same boundary conditions, but with a redistributed internal force flow.

Various polyhedral subdivision schemes can be used to subdivide the internal space of the force polyhedrons. In general, any cell decomposition of the global force polyhedron that is closed and has planar faces, can represent the equilibrium of a spatial funicular form. Consider the funicular form and its global force tetrahedron of Figure 2. Recursive barycentric subdivision of the global force tetrahedron results in various structural forms preserving the same boundary conditions (Figure 4).

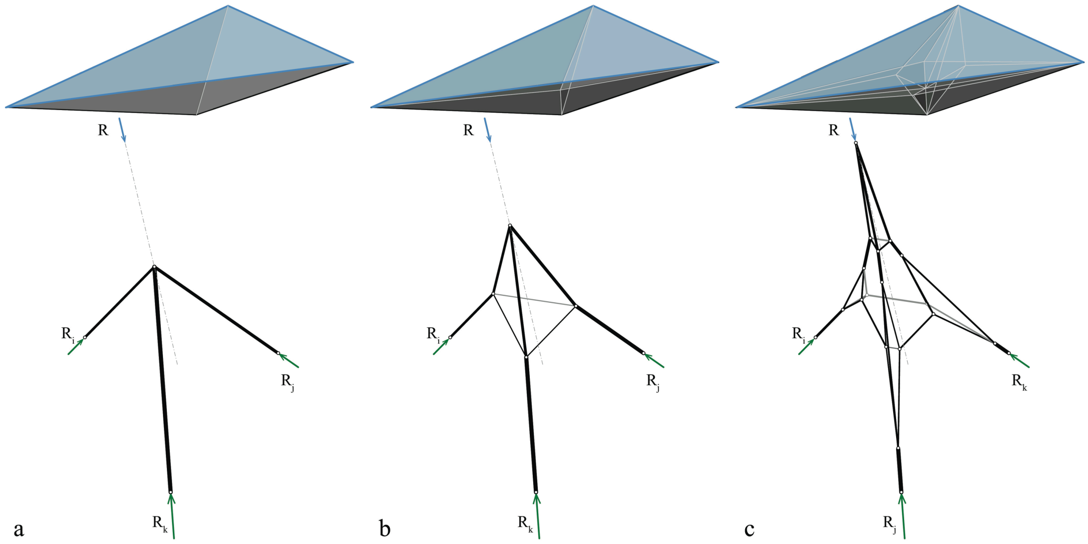

Figure 4: Three steps of barycentric subdivision of the global force tetrahedron.

More complex, spatial funicular forms can be derived by subdividing not only the external force polyhedron, but also the internal cells of the force diagram. Consider the form and force diagrams of the compression-only branching structure in Figure 5. Subdividing the internal cells of the force polyhedron results in a spatial form that is topologically different from the original branching structure. However, it preserves the boundary conditions of the initial form.

Polyhedral subdivision of the force diagram not only provides a variety of design possibilities for given boundary conditions, but also increases the number of members in the structural form, often reducing the magnitude of forces carried by each element. Therefore, this approach can be used as a strategy to deal with buckling due to excessive axial forces in the members of a structure.

Figure 5.a shows the exploded axonometric view of the force diagram and its reciprocal branching structural form. The exploded force diagram shows multiple layers of polyhedral cells and the external force polyhedron. The area of the highlighted faces in the force diagram represents the magnitude of force in the highlighted, corresponding members of the form. Figure 5.b shows a subdivided instance of the force polyhedron of Figure 5.a and its corresponding form diagram. Note that the number of internal faces in the force diagram has been increased as well as the number of elements in the form diagram. However, the force magnitude is significantly reduced in the newly created members. Comparing the force magnitudes in edges e_i in the original form diagram and its subdivided version shows that the magnitude of forces has been approximately reduced by a third as a result of the subdivision process.

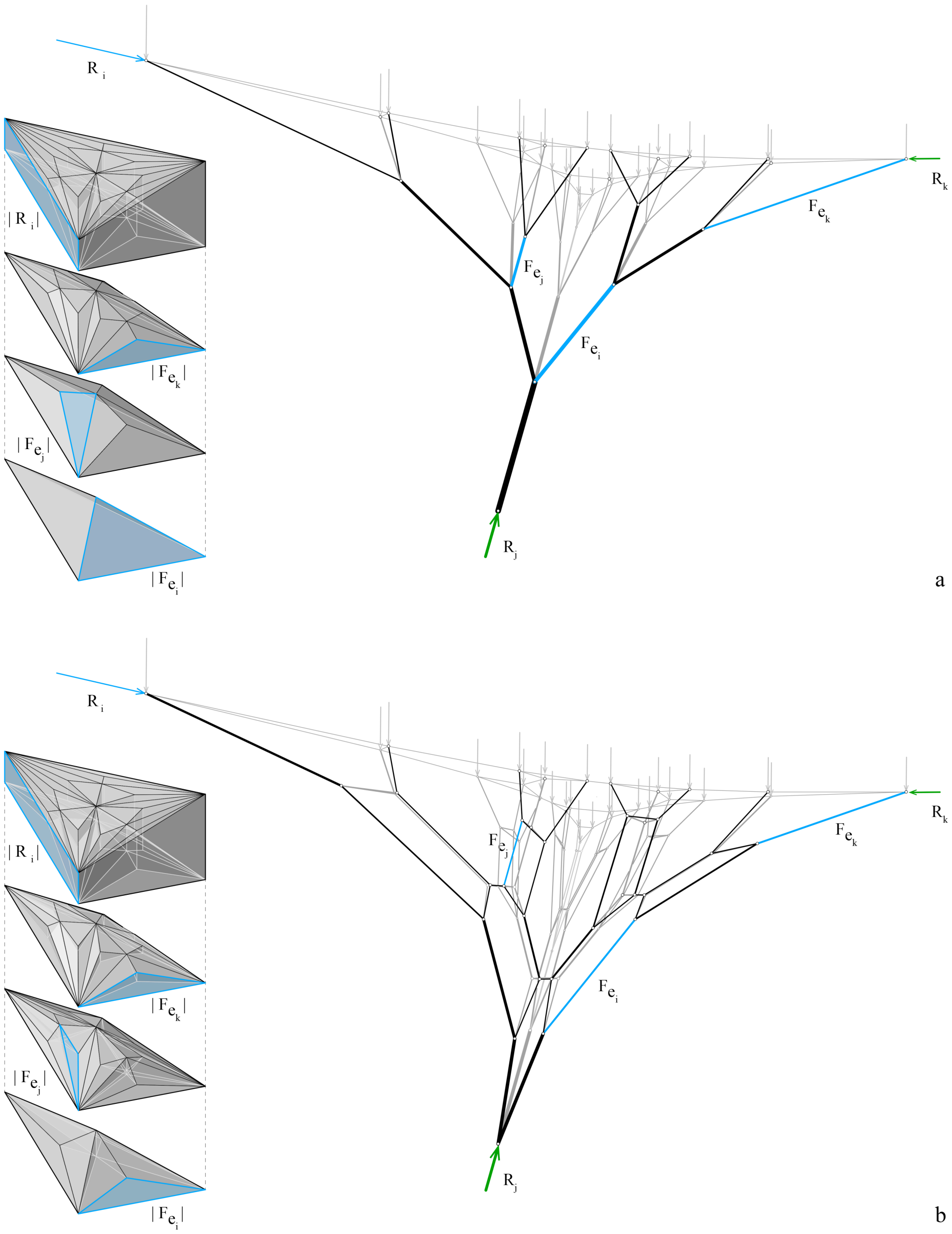

Figure 5: The exploded force diagram and corresponding branching form diagram, a) before and b) after subdivision. The areas of highlighted faces in the polyhedral force diagram represent the magnitude of forces in the highlighted edges of the form diagram.

3. Conclusion and Discussion

This paper provided a new design methodology for deriving 3D funicular structural forms by subdividing the force polyhedron. It mainly addressed two methods of subdividing the global force polyhedron: subdivision of the face corresponding to the resultant force, and subdivision of the internal space of the force polyhedron without changing the external faces. The force diagram subdivision strategy in design provides the opportunity to develop a variety of equilibrium solutions without changing the boundary condition. Subdivision of the force polyhedron can be used to decrease the magnitude of forces in members of the form. Therefore, this method can be used as a strategy in dealing with buckling in spatial funicular structures, a topic which will be developed in future research.

References

[1] Akbarzadeh M., Van Mele T. and Block P., Equilibrium of spatial structures using 3D reciprocal diagrams. *Proceedings of IASS Symposium 2013*, Wroclaw, Poland, 2013.

[2] Akbarzadeh M., Van Mele T. and Block P., Compression-only form finding through finite subdivision of the external force polygon. *Proceedings of the IASS-SLTE Symposium 2014*, Brasilia, Brazil, 2014.

[3] Akbarzadeh M., Van Mele T. and Block P., On convex polyhedral force diagrams and the equilibrium of compression or tension-only spatial systems of forces. *Computer-Aided Design*, 2015; **63**; 118-128.

[4] Akbarzadeh M., Van Mele T. and Block P., 3D Graphic Statics: Constructing global equilibrium. *Proceedings of the IASS Symposium 2015*, Amsterdam, Netherlands, 2015.

[5] Allen E. and Zalewski W., *Form and Forces: Designing Efficient, Expressive Structures*. John Wiley Sons, New York, 2010.

[6] Beghini L. L., Carrion J., Beghini A., Mazurek A. and Baker W. F., Structural optimization using graphic statics. *Struct Multidisc Optim*, 2013; **49**; 351-366.

[7] Block P., *Thrust Network Analysis: Exploring Three-dimensional Equilibrium*. PhD thesis, MIT, Cambridge, MA, USA, 2009.

[8] Lachauer L. and Block P., Compression Support Structures for Slabs, *Proceedings of Advances in Architectural Geometry,* Paris, France, 2012.

[9] Rankine M., Principle of the equilibrium of polyhedral frames. Philosophical Magazine, 1864; **27**(180); 92.

[10] Warren J. and Weimer H., *Subdivision Methods for Geometric Design: A Constructive Approach*. Morgan Kaufmann, 2001.