



# Lightweight Structures and the Geometric Equilibrium in Dragonfly Wings

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## Abstract

This research investigates the use of graphic statics in analyzing the structural geometry of a natural phenomenon to understand its performance and its relevant design parameters. Nature has always been the source of inspiration for designers, engineers, and scientists. Structural systems in nature are constantly evolving to optimize themselves with their boundary conditions. This optimization follows certain design rules that are quite challenging for a human to formulate or even comprehend. A dragonfly wing is an instance of a high-performance, lightweight structure that has intrigued many researchers to investigate its geometry and its performance as one of the most light-weight structures designed by nature. There are extensive geometrical and analytical studies on the pattern of the wing, but the underlying design logic is not clear. The geometry of the internal members of the dragonfly wings mainly consists of convex cell which may represent a compression-only network on a 2D plane. However, this property has not been geometrically analyzed from this perspective to confirm the hypothesis. In this research, we use the methods of 2D graphic statics to construct the force diagram from the given structural geometry of the wing. We use algebraic and iterative methods to report the topological and geometric properties of the form and force diagrams such as the degrees of indeterminacies of the network. For sample wings, we separate the internal and the boundary edges, construct the force diagram, and finally reconstruct the structural forms. Comparing the magnitude of the forces of the reconstructed network with the actual structure of the wing using the edge lengths of the force diagram will shed light on the performance of the structure. Multiple analytical studies will be provided to compare the results in both synthetic and natural networks and drive solid conclusions. The success in predicting the force flow in the natural structural pattern using graphic statics will expand the use of these powerful methods in reproducing the similar geometry of the natural structural system for the use in many engineering and scientific problems. It will also ultimately help us understand the design parameters and boundary conditions for which nature produces its master-pieces.

**Keywords:** graphic statics, form and force diagrams, natural structures.

## 1. Introduction

### 1.1. Previous Research on Dragonfly Wings

Nature has always been the source of inspiration for designers, engineers, and scientists. Among a variety of natural structures, a dragonfly wing is an instance of a high-performance, lightweight structure that has intrigued many researchers to investigate its geometry and its performance to be used in bio-inspired design. The previous research related to the dragonfly wings falls into three main categories -- the description, the measurement, and the design.

For description, researchers use high-tech tools to observe the dragonfly wings and summarize the properties (usually morphology or pattern). Related research includes: 1) the nano-scale observation of the wings, for example, the description of the nanomechanical behavior of the dragonfly wings [1], and the finite element analysis of the structure and nano-mechanical properties of dragonfly wings [2]; 2) the detail modelling and scanning of the joints, for example, the effect of the resilin-containing vein joints on the stress distribution within the dragonfly wings [3], and three-dimensional finite-element models of vein joints [4]; and 3) the general description of the morphology information, for example, the insect wing mechanical design by comparing different insect wings including dragonfly wings [5], the modeling and fabrication of the dragonfly wings patterns with the morphology analysis [6], the dynamic deformation of the insect wings [7], and the morphometric measurements of scanned images of dragonfly wings [8].

The other aspect focuses on the measurement and assessment of the dragonfly wings under certain conditions. It includes: 1) aerodynamic performance measurement, for example, the aerodynamic data collecting and presenting through wind-tunnel test [9], the effectiveness and efficiency of two pairs of wings when flying [10], and the aerodynamic flow visualization when flying [11]; and 2) the other functional performance, for example, the measuring the natural frequencies and mode shapes of dragonfly wings [12], the superhydrophobicity and self-cleaning properties of the dragonfly wings in nano scale [13], the functional characteristics of the dragonfly wings [14], and the initiation and progression of wing damage [15].

The last aspect applies the design ideas inspired from the dragonfly wings, for example the design of new materials and new structures. Related research includes: 1) the design of flight vehicle with flapping wings inspired from the dragonfly wings [16], and 2) the exploration of the dragonfly wings in nanoscale and the design of the nano-pattern of medical implant material inspired by the dragonfly wings [17].

## **1.2. Problem Statement and Objectives**

In the previous research, the logic follows three steps: 1) measuring the dragonfly wings in different scales; 2) testing the performance of the dragonfly wings under different conditions; 3) mimicking the dragonfly wings for design solutions in similar environments. However, there is an obvious missing logic between steps 2 and 3. Previous research only describes the property and the functions of dragonfly wings. The intrinsic logic and the explanation, for example, why nature designs dragonfly wings in such a way, are not provided. In addition, in the design-related research, the patterns of the dragonfly wings are directly used as the design cases, without meeting the specific requirements, for example the boundary conditions. Thus, the application of the dragonfly-wing-inspired design remains superficial at mimicry level.

Based on the above analysis, we propose the following hypothesis: the geometry of the internal members of the dragonfly wings mainly consists of convex cells, which may represent a compression-only network [18, 19] on a 2D plane. This property motivates us to analyze the dragonfly wings (Figure 1) from the perspective of axial loading. Therefore, this research aims at the investigation of the use of geometric-based structural design method (graphic statics [20, 21, 22, 23, 24, 25, 26, 27, 28, 29]) in analyzing the structural geometry of natural phenomenon to understand the internal flow of force and their relevant design parameters, thus finally proving that this geometric-based structural design method can be used to as a form finding tool for the design of similar structures.

## **2. Methodology**

### **2.1. Graphic Statics Method**

Graphic statics is a geometry-based method, where the equilibrium of force is shown by geometric diagrams. In 2D graphic statics, the force diagram shows the force magnitude of the corresponding form diagram. The length of each edge in the force diagram represents the force magnitude of the

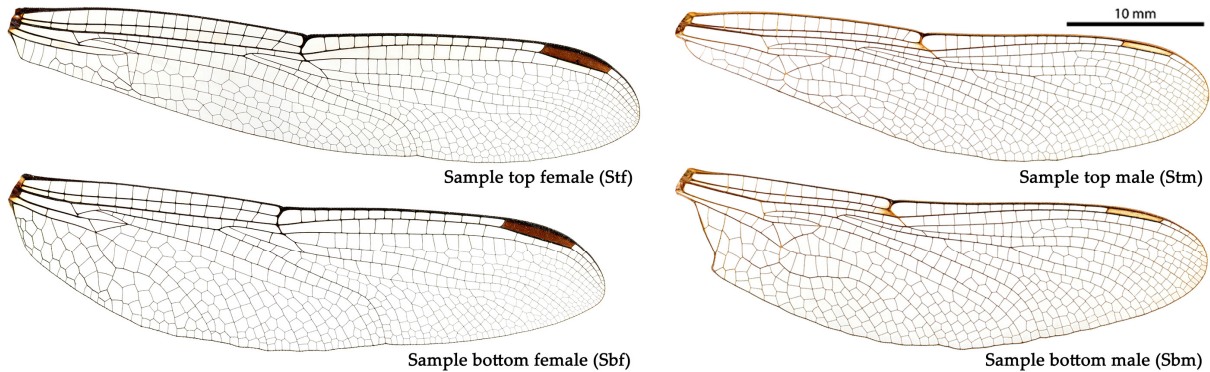


Figure 1: Dragonfly wing samples and their patterns [30].

corresponding edge in the form diagram. Figure 2a shows a structural network and its force diagram. The geometric transformation between the force diagram and the form diagram can be achieved using graphic statics method. Figure 2b shows a 3D network and its projection and reciprocal diagram. According to the force magnitude, the structural thickness of the form can be predicted.

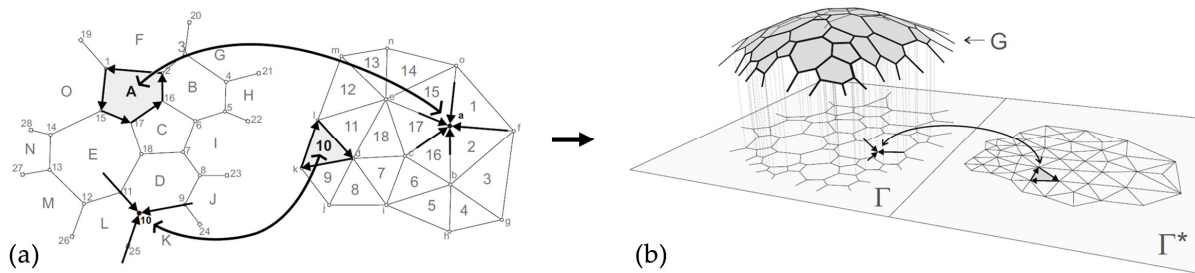


Figure 2: Thrust Network Analysis: (a) two-dimensional equilibrium: a structural network and its force diagram; and (b) three-dimensional equilibrium: a 3D structural network and its projection and reciprocal diagram [31].

In the case of the dragonfly wing, its 2D pattern can be regarded as a structural form (Figure 3a), which contains a group of closed polygons and each edge of the polygons has a unique structural thickness. The simplified version of the dragonfly wing (Figure 3b) contains only the geometric information, it is a representative form diagram of the dragonfly wing. Using graphic statics method, its corresponding force diagram (Figure 3c) can be generated. Thus, by mapping the force magnitude to the structural thickness, the corresponding structural form can be re-generated. The goal of this research is to compare our generated wings with the real wings, and show whether the force distribution in the real wing follows the rules of graphic statics. Therefore, the force diagram can explain the intrinsic logic of the design of the dragonfly wing by nature.

## 2.2. Compression-only Network

To better use graphic statics to extract the force diagram, we assume the entire dragonfly wing as a self-stressed system, in which the internal members bear axial compression forces, while the boundary members and the membrane (abstractly rebuilt as triangulated members) together bear tension forces as a constraint (Figure 4). Therefore, the internal structure represents a compression-only form, and the rest represents a tension-only structure. By the following experiments, we will show why this assumption is correct and how it benefits to the structural analysis.

Since we do not have a clear understanding of the external force of the dragonfly wing, we first exclude the boundary edges of the dragonfly wing and focus on the internal network of the edges. Figure 5 shows the process of the data preparation. The original dragonfly wing image (Figure 5a) is transformed into

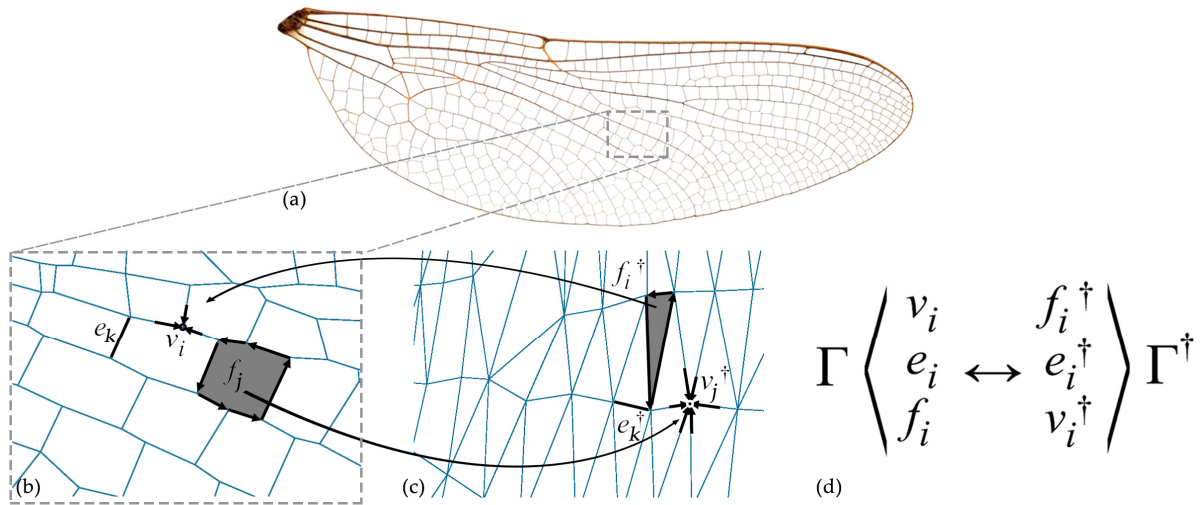


Figure 3: The force and form of the internal structure of the dragonfly wings: (a) the actual form of the entire wing; (b) form diagram containing vertices ( $v$ ), edges ( $e$ ), and faces ( $f$ ); (c) force diagram containing vertices ( $v^\dagger$ ), edges ( $e^\dagger$ ), and faces ( $f^\dagger$ ); and (d) the topological relationship between the elements.

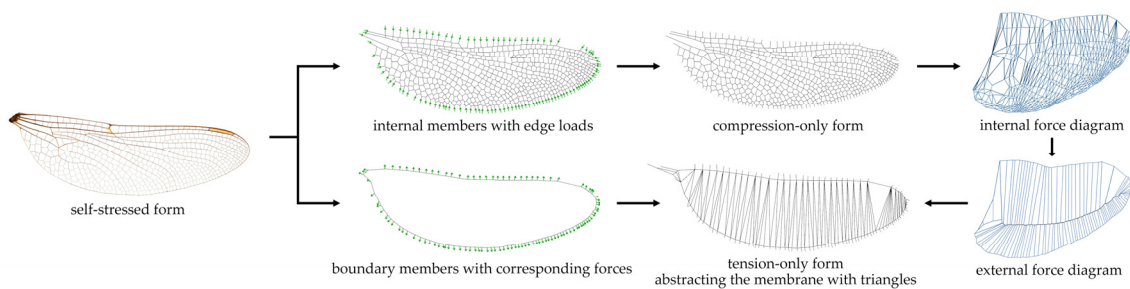


Figure 4: The loading scenario of the dragonfly wing: entire wing as a self-stressed system, internal members as a compression-only form, boundary members and the membrane as a tension-only form.

the vector-based geometry with convex and non-convex polygons (Figure 5b) using image processing techniques. The non-convex polygons (Figure 5c) are slightly adjusted to make convex polygons (Figure 5d). This allows the graphic statics method to work on the compression-only network. By removing the boundary edges, the internal form (Figure 5e) is generated.

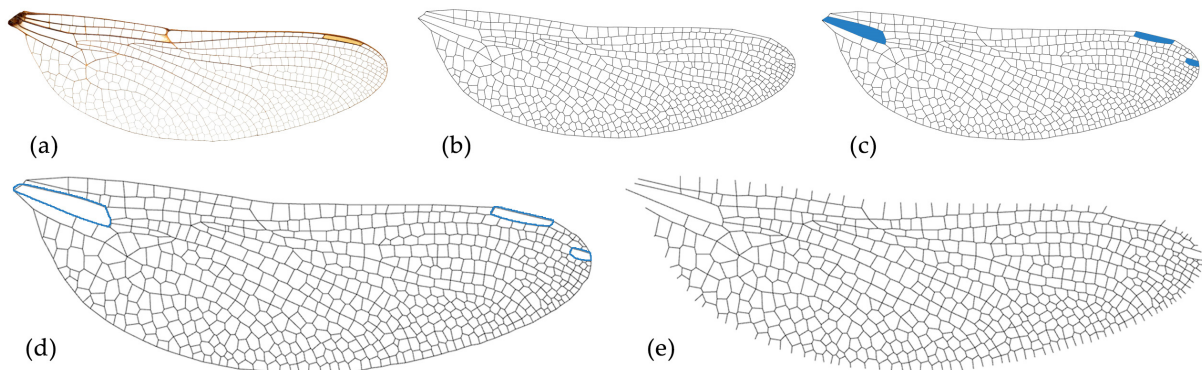


Figure 5: Data preparation: (a) original dragonfly wing image; (b) vectorized form; (c) modifying non-convex cells (3 out of 573 (0.52%) cells); (d) modified convex form; and (e) internal form.

### 2.3. Algebraic Solution

The next step is to generate the force diagram based on the internal form diagram. In 2D graphic statics, each edge in the force diagram is perpendicular to the corresponding edge in the form diagram. Figure 6a shows a form diagram, which contains six internal edges and six external edges. In Figure 6b, the proposed edges are presented as the dark blue lines, which are perpendicular to the edges in Figure 6a. Therefore, each vertex ( $v_i$ ) in the form diagram has a dual polygon ( $f_i^\dagger$ ) in the force diagram. For each polygon ( $f_i^\dagger$ ), a group of equations can be derived based on its edges ( $e_i^\dagger$ ) (Figure 6c).

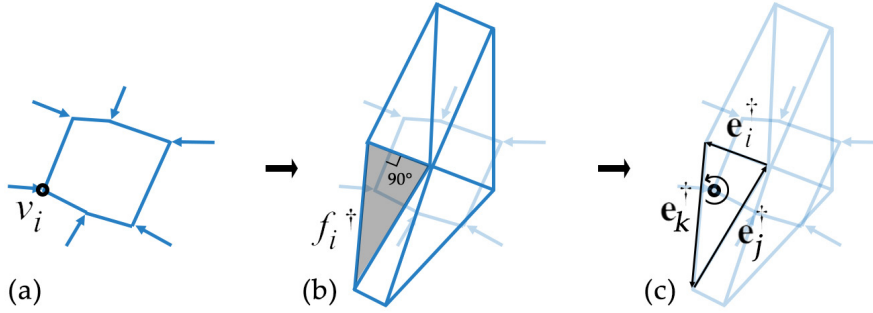


Figure 6: Algebraic solution: building equations.

Indeed, since the face  $f_i^\dagger$  is a closed polygon, the sum of its edge vectors  $e_j^\dagger$  should be zero. Hence, we obtain a vector equation

$$\sum_{e_j^\dagger} u_j^\dagger q_j = 0$$

where the sum runs over the edges  $e_j^\dagger$  of the face  $f_i^\dagger$ ,  $u_j^\dagger$  denotes the unit edge vector of the edge  $e_j^\dagger$ , and the  $q_j$  are the variables representing the lengths of the edges  $e_j^\dagger$ . Writing these equations around every face  $f_i^\dagger$  provides a linear equation system for the edge length vector  $q$  which can be described by a  $[2v * e]$  matrix, called the equilibrium matrix

$$Aq = 0$$

If the equilibrium matrix  $A$  is of full rank (or the Geometric Degrees of Freedom (GDOF) is zero), then the only solution is the zero vector. In this case, the force diagram collapses to a point.

### 2.4. Iterative Solution

However, after experiments, we found that most of the dragonfly wings in our dataset have a GDOF of 0, which means there are no accurate solutions for most of the cases. Thus, the algebraic solution is not stably applicable in all the cases of the dragonfly wings.

Therefore, we present an iterative method of PolyFrame [32, 33] that can generate the geometries of the force diagram with a deviation (user-defined tolerance). The algorithm starts from constructing the graph of the force diagram with incorrect edge lengths. Each polygon ( $f_i$ ) in the form (Figure 7a) is transformed into a vertex ( $v_i^\dagger$ ) in the force (Figure 7b). Then the iterative method is applied to optimize the position of each vertex ( $v_i^\dagger$ ) to make the corresponding edges perpendicular (Figure 7c). We also define the difference between the vertical angle ( $90^\circ$ ) and the angle of the two corresponding edges ( $\alpha$ ) as the deviation ( $\delta$ ). The optimization process works by a loop to minimize the value of  $\delta$  (Figure 8). With this geometric method, we can generate an approximate solution of the force diagram.



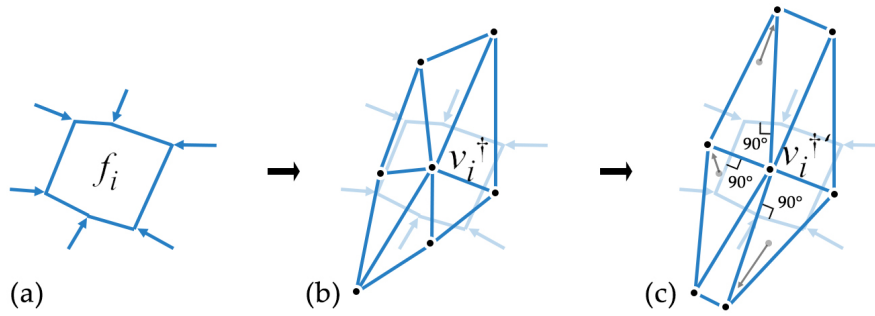


Figure 7: Geometric solution: iterative method to update the positions of vertices.

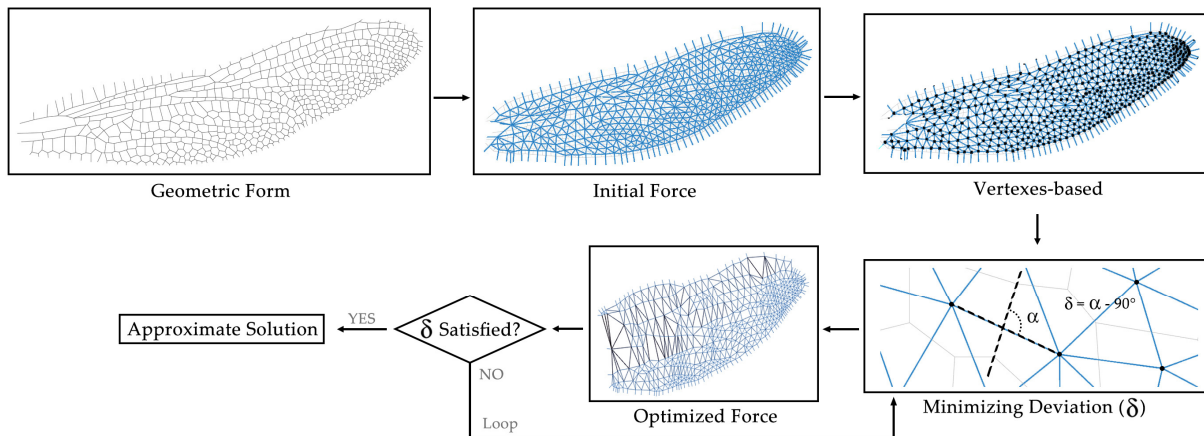


Figure 8: Flow chart of geometric method.

### 3. Results

#### 3.1. Internal Form and Force

Using graphic statics method, Figure 9 shows the force diagram generated from the form diagram of the dragonfly wing (form-to-force), and Figure 10 shows the generation of the form diagram from the generated force diagram (force-to-form). The maximum deviations for the two processes are  $46.64^\circ$  and  $20.78^\circ$ , while the average deviations for the two processes are  $2.64^\circ$  and  $0.64^\circ$ . Therefore, the geometric method successfully transforms the force diagram and the form diagram with small values of average deviations. Figure 11 shows the distribution of the deviation of the form-to-force and the force-to-form generation. Most of the members have a deviation smaller than  $1^\circ$ , and there are decreasing numbers of members as the deviation increases. Only a few members have a deviation larger than  $20^\circ$  in the form-to-force process and  $10^\circ$  in the force-to-form process. Therefore, most of the members in the force diagram are accurately generated according to the corresponding form diagram, with the meaning of the internal force magnitude.

#### 3.2. The Thickness of the Dragonfly Wings Members

With the force diagram, the internal force for each form member can be obtained, thus the structural thickness can be inferred, which is proportional to the force magnitude. We first measure the thickness of the real wing in pixel, the minimum and maximum widths are approximately 6 pixels and 60 pixels. We found that the ratio of the minimum and maximum force magnitude is also approximately 1:10, thus we remap the force magnitude into the range of 3 to 30, and generate the structural pipes with the radius of the remapped values. Therefore, the predicted internal structure (Figure 12a) is generated. Figure 12 shows the comparison of the generated internal structure and the real internal wing.

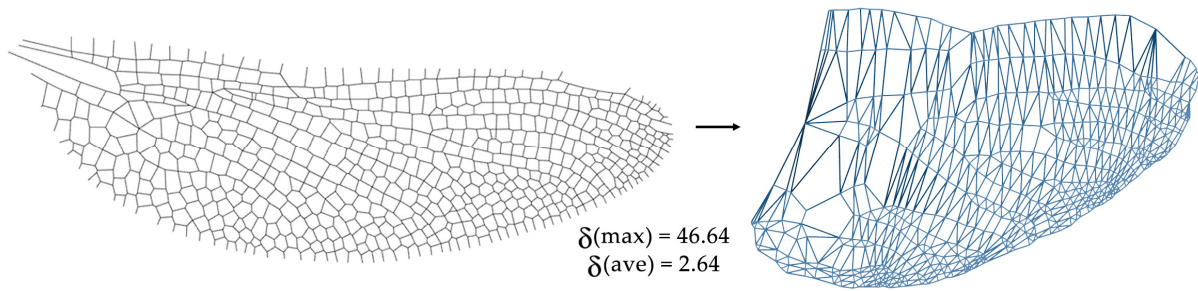


Figure 9: The maximum and average form-to-force deviations in degrees ( $\delta$ ) in the generation of force from the form.

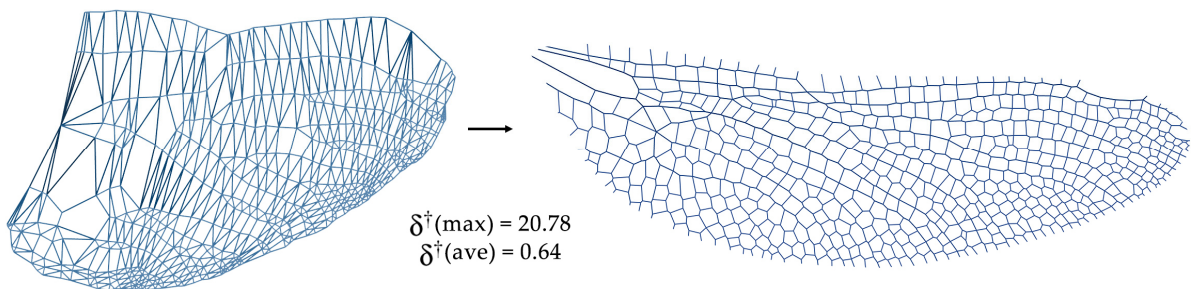


Figure 10: The maximum and average force-to-form deviations in degrees ( $\delta^\dagger$ ) in the generation of form from the force.

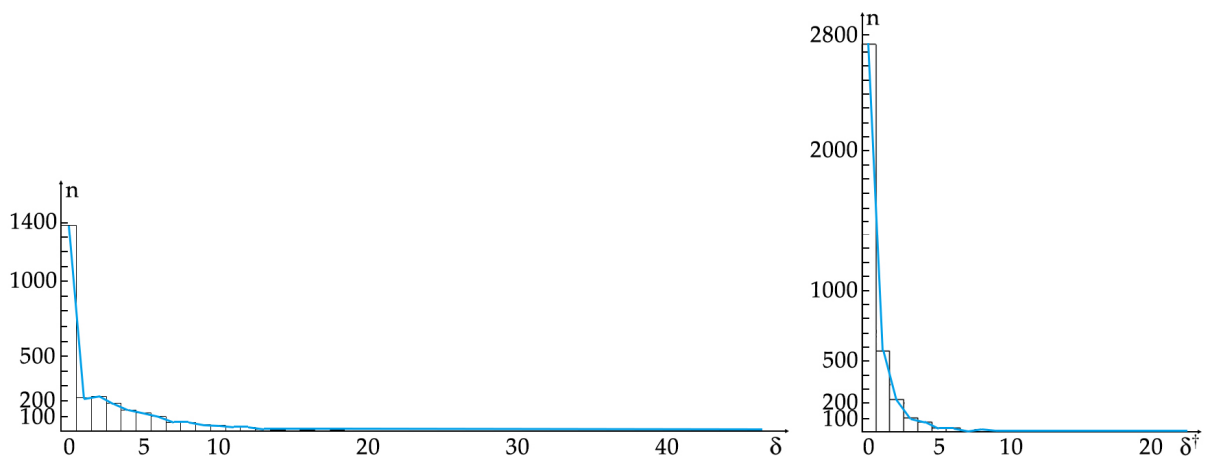


Figure 11: The distribution of two deviations.  $\delta$ : form-to-force deviation in degrees.  $\delta^\dagger$ : force-to-form deviation in degrees.  $n$ : number of members.

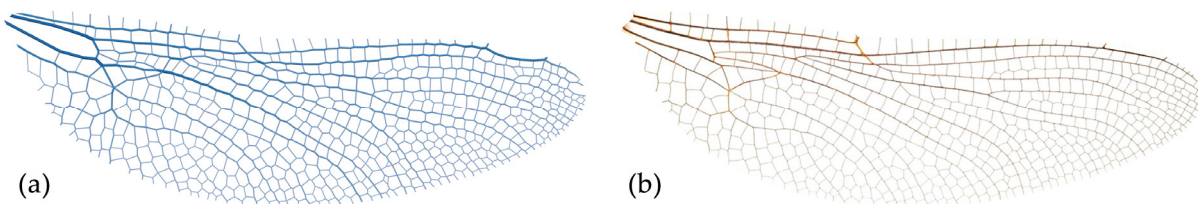


Figure 12: Internal structure: (a) generated, the thickness ranges from 3 to 30 units; and (b) original.

In order to compare the similarity of the generated and the real internal wing, we define the accuracy measure ( $\theta$ ) as the following process (Figure 13). First, the radius of each structural member is extracted as  $r_i$ , and the normalized radius  $R_i$  is calculated (Equation 1), thus the value range of  $R_i$  is 0 to 1. Second, for each member in the real and the generated wings, the accuracy  $\theta_i$  is calculated based on the absolute error of these two values (Equation 2). Last, the overall accuracy  $\theta$  is the weighted average of all  $\theta_i$  based on the volume  $V_i$  of each member (Equation 3).

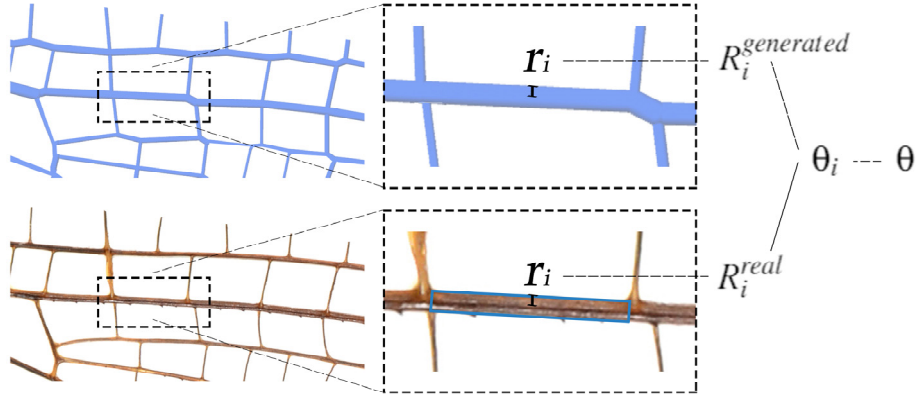


Figure 13: The calculation of the accuracy ( $\theta$ ).  $r_i$ : radius;  $R_i$ : normalized radius;  $\theta_i$ : accuracy for each member; and  $\theta$ : overall accuracy.

$$R_i = \frac{r_i - \min(r)}{\max(r) - \min(r)} \quad (1)$$

$$\theta_i = 1 - |R_i^{real} - R_i^{generated}| \quad (2)$$

$$\theta = \frac{\sum_{i=1}^N \theta_i \cdot V_i}{\sum_{i=1}^N V_i} \quad (3)$$

Figure 14 shows the generated and the real internal wings in our testing dataset, and Table 1 shows their statistic values of the deviation and the accuracy. According to the statistics, in the force-to-form generation, all the six testing wings have an average deviation of  $2.77^\circ$ , and 78.83% of the members have a deviation smaller than  $5^\circ$ . In the form-to-force generation, the average deviation is  $0.97^\circ$  and 97.62% of the members have a deviation smaller than  $5^\circ$ . The overall accuracy is 91.9%.

### 3.3. Summary of the Workflow

Based on the above observation, we conclude the workflow as Figure 15 shows. First, the structural thickness is partially removed from the original dragonfly wing image, with only the geometry of the wing left. Then, our graphic statics method transforms the geometry of the form into the geometry of the force. By mapping the edge lengths in the force geometry to the structural thickness of the edge in the form geometry, the wing is generated again with the predicted structural thickness. If the accuracy of the comparison of the real and the generated wings is high as expected, our assumptions are then proven from the aspect of structural engineering.

Therefore, through the comparison of more pieces of dragonfly wings, we show the evidence that our assumptions are correct. The high accuracy of the comparison of the real and the generated wings proves our assumptions: 1) graphic statics method can be used to analyze the dragonfly wings structures; 2) the intrinsic logic of the structural characteristics of the dragonfly wing can be represented as the force diagram.



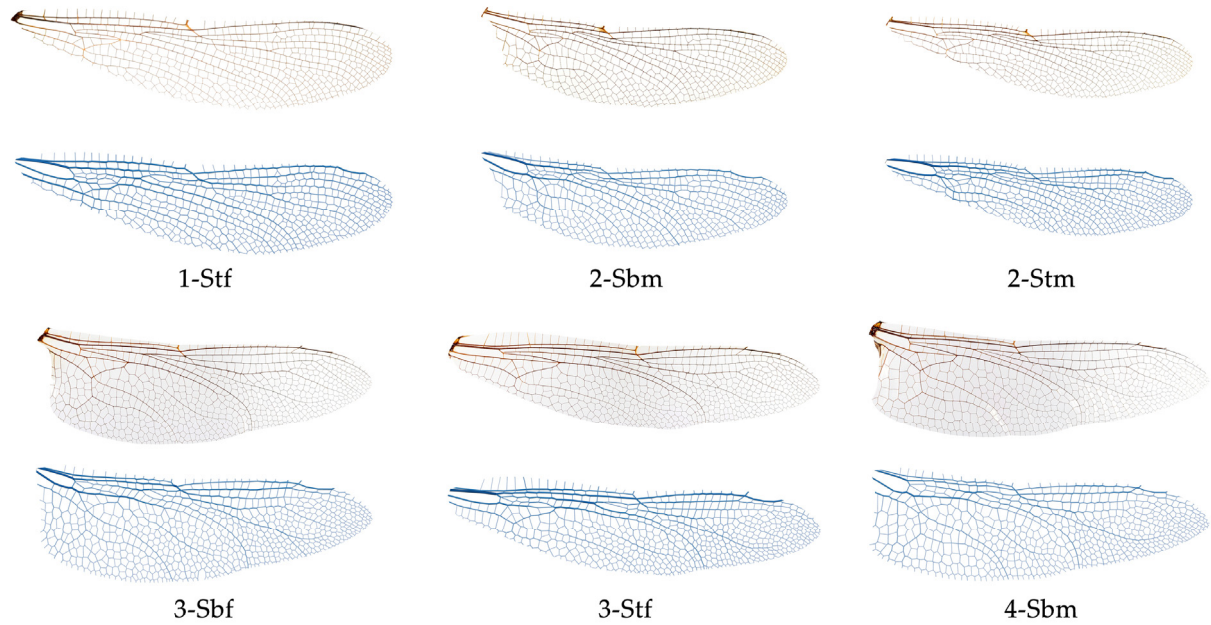


Figure 14: Comparison of the original and the generated structural thickness (internal only).

Table 1: Deviation and accuracy table for Figure 14:  $\delta$ : form-to-force deviation in degrees;  $\delta^\dagger$ : force-to-form deviation in degrees; and  $\theta$ : accuracy.

Sample	$\delta(\max)$	$\delta(\text{ave})$	$\%(<5^\circ)$	$\delta^\dagger(\max)$	$\delta^\dagger(\text{ave})$	$\%(<5^\circ)$	$\theta$
1-Stf	52.26	2.37	0.83013	21.54	0.57	0.98245	0.922349
2-Sbm	44.76	2.72	0.78754	17.44	0.69	0.97179	0.922321
2-Stm	42.04	2.57	0.82237	42.68	0.69	0.97446	0.935846
3-Sbf	55.61	2.81	0.78835	38.69	0.68	0.98546	0.891284
3-Stf	35.35	3.13	0.76083	30.45	0.68	0.98846	0.923929
3-Sbm	38.43	3.01	0.76544	29.68	0.70	0.98057	0.917173
Average	44.74	2.77	0.78833	30.08	0.67	0.97624	<b>0.918817</b>

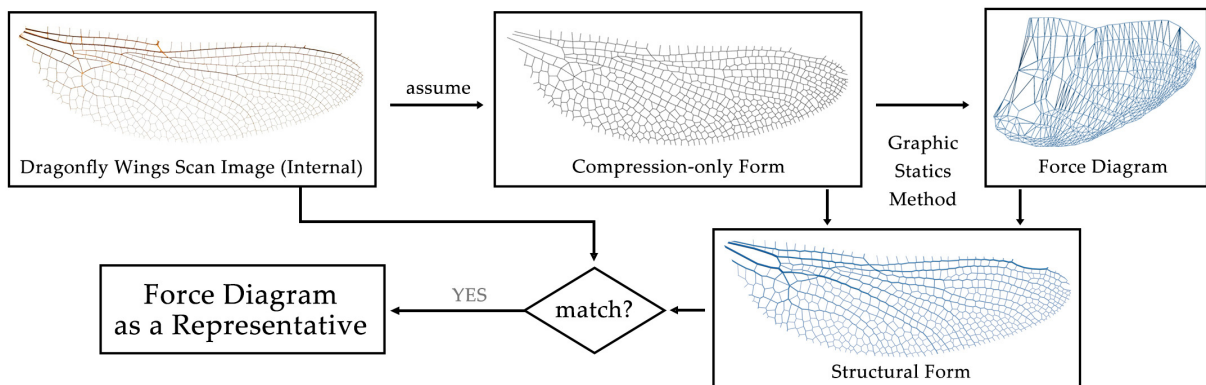


Figure 15: Flow chart of predicting and analyzing the thickness of the dragonfly wings members.

#### 4. Conclusion and Future Outlook

In this paper, we show a graphic statics method to transform the geometries between the form diagram and the force diagram of the internal structures of the dragonfly wing. By generating the force diagram and measuring the force magnitude, the structural thickness can be predicted. The structural form generated by our method and the real one is highly similar, which proved that our graphic statics method can be used to analyze the internal structural form of the dragonfly wing.

As a secondary conclusion, this paper also indicates that the force diagram is another representative of the dragonfly wing besides the form diagram. It reveals the intrinsic logic of the generation of the geometries and the thickness of the dragonfly wing. Therefore, the force diagram contains structural information that is also helpful for further research on the dragonfly wing structures.

However, the external loading scenario is not clear in this research, including the external forces from the boundary ring and the membrane as a tension-only network. Combining the external force diagram with the internal force diagram will be one aspect of the future research. Besides, the future research of this paper will also involve advanced analysis and generation techniques, for example machine learning, to further study the design from nature, from the perspective of structural analysis of the force diagram.

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