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Generating and Optimizing a Funicular Arch Floor Structure

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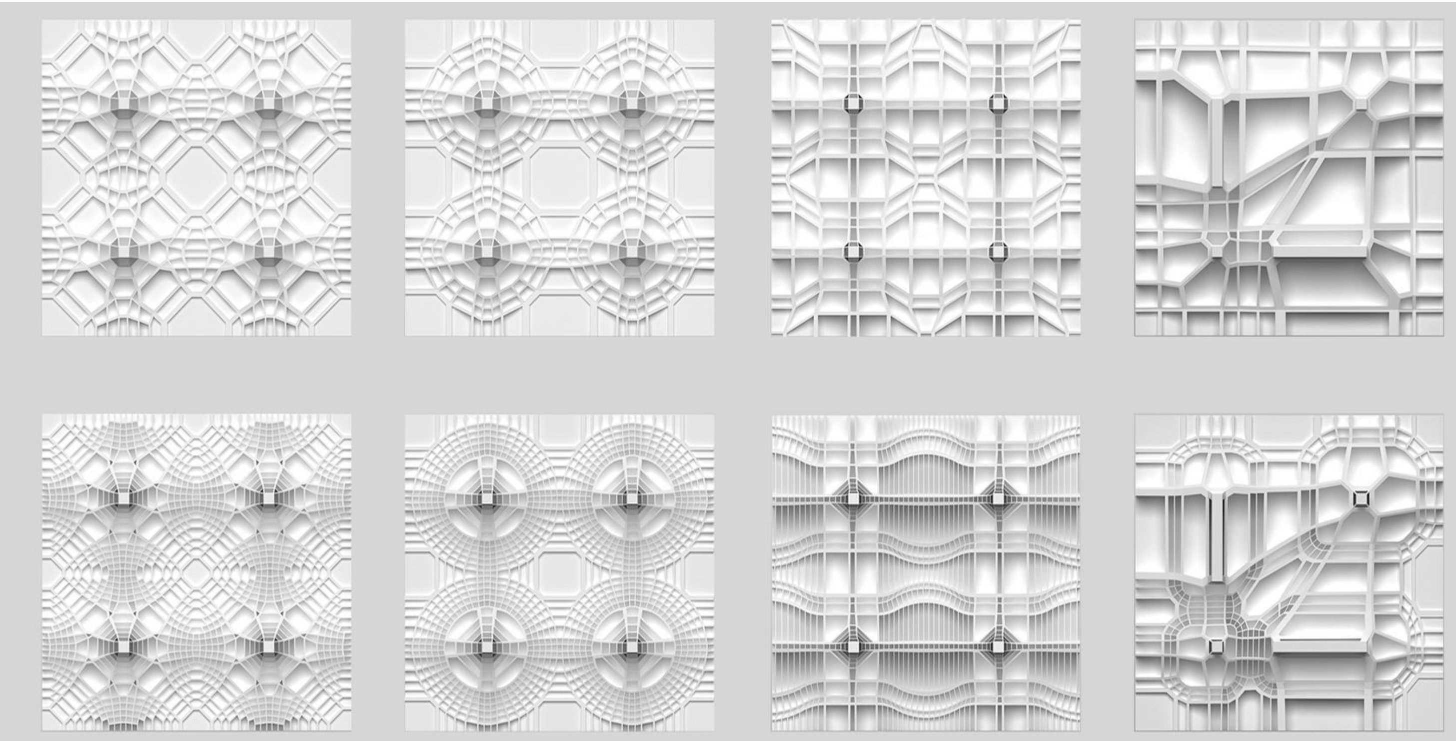

Hero image: Different structural forms generated by graphic statics.

ABSTRACT

In this paper, we propose a geometry-based generative design method to generate and optimize a floor structure with funicular building members. This method challenges the antiquated column system, which has been used for more than a century. By inputting the floor plan with the positions of columns, designers can generate a variety of funicular supporting structures, expanding the choice of floor structure designs beyond simply columns and beams and encouraging the creation of architectural spaces with more diverse design elements. We further apply machine learning techniques (artificial neural networks) to evaluate and optimize the structural performance and constructability of the funicular structure, thus finding the optimal solutions within the almost infinite solution space. To achieve this, a machine learning model is trained and used as a fast evaluator to help the evolutionary algorithm find the optimal designs. This interdisciplinary method combines computer science and structural design, providing flexible design choices for generating floor structures.

Figure 1. History of concrete structure roof. Image credit to the designers. From top left to bottom right: a) Riverside Church; b) Lanificio Gatti; c) Palazzo del Lavoro; d) floor design from Block Research Group; e) Zoology Lecture Hall at the University of Freiburg; f) Dfab House.

INTRODUCTION

Background

The Maison Domino was invented more than 100 years ago by Le Corbusier (Anderson 1984) and is still widely used in open floor-plan designs. The column supports the beam while the beam supports the ceiling, which covers the main space.

However, with the development of computational design techniques, a new possibility is emerging: A funicular arch structure can replace columns and beams. Recent developments and explorations in funicular arch structure from Rafael Guastavino, floor design from Philippe Block (Adriaenssens, Block et al. 2014), and reinforced concrete from Luigi Nervi have brought out new aesthetics and the possibilities of more economical structure systems (Figure 1).

Graphic Statics

In the design of the funicular arch structure, the graphic statics method is widely used to evaluate and generate the force and form. Graphic statics (2D/3D) is a geometry-based structural design and analysis method. The history of graphic statics can be traced back to the Hellenistic Age, when Archimedes used algebraic formulas and illustrations to explain in his book On the Equilibrium of Planes that the weight of an object is inversely proportional to the distance under equilibrium conditions in the law of levers.

The Renaissance was the beginning of modern mechanics. Galileo Galilei, Robert Hooke, and Isaac Newton made great contributions to the scientific development of mechanics. Specific to graphic statics, mechanics contains three important factors: Forces are represented as vectors, forces can be composed and decomposed, and a balance of forces can be achieved under equilibrium conditions. In 1586, Simon Stevin proved the parallelogram rule of the decomposition and synthesis of forces with the load test on the inclined plane, pioneering the use of geometry to find the equilibrium of forces (Stevin 1586). In 1864, after systematically reviewing and expanding the field's knowledge, Karl Culmann named this subject "graphische Statik" (graphic statics) in his book Die Graphische Statik (Culmann 1864), which was widely accepted by the academic community. Graphic statics was then formally established with the successful follow-up research in 2D graphic statics (Maxwell 1864, Maxwell 1870, Bow 1873, Cremona 1890).

However, 2D graphic statics has its own limitations (Akbarzadeh 2016); only 2D abstractions of 3D structures could be designed, although Culmann also proposed a 3D solution of graphic statics (Culmann 1864), which was never explained and proved in detail in his book. Maxwell applied this 3D method to a specific case of geometric operation (Maxwell 1864), but the complex calculations stopped him from further research in 3D graphic statics, and this theory has been left intact since 1864.

But with the recent development of computing power, the complex geometric calculations of 3D graphic statics can be now performed through digital computation. Thus, 3D graphic statics has attracted the attention of researchers again. Aided by computers, architects developed digital algorithms to generate 3D forms from 3D force diagrams (Theodoropoulos 2000, Block and Ochsendorf 2007, Van Mele, Lachauer et al. 2012, Fivet and Zastavni 2013, Stouffs, Janssen et al. 2013, Van Mele and Block 2014, Akbarzadeh, Van Mele et al. 2015, Bolhassani, Akbarzadeh et al. 2018).

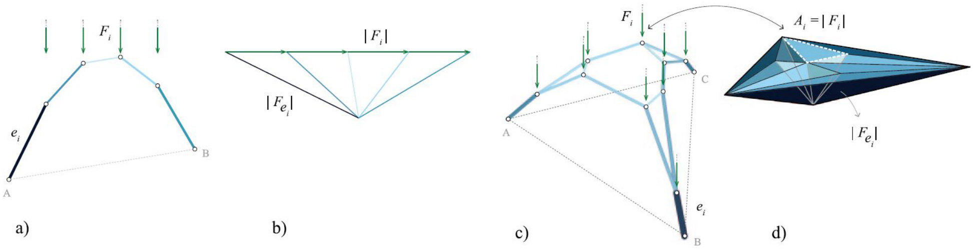

Figure 2. 2D versus 3D funicular solutions and their corresponding force diagrams, by Akbarzadeh et al. (2016)

The computational solution of 3D graphic statics helps designers generate 3D polyhedral forms by manipulating force diagrams with given boundary conditions.

In the form finding of 3D graphic statics, the transformation rules from force diagrams to form diagrams work much as they do in 2D. Figure 2 shows the comparison of 2D and 3D graphic statics, where Figures b) and d) are the force diagrams and Figures a) and c) are the form diagrams. Each applied load “Fi” in the force diagram represents a corresponding load force “Fi” in the form diagram, with the two perpendicular to each other. Each exterior supporting force “Fei” in the force diagram results in a structural member “ei” in the form diagram, which shows the corresponding form of a force diagram. But the difference is that, in 2D graphic statics, the forces are drawn as lines, while, in 3D graphic statics, the forces are represented as planar surfaces, so the forces and the corresponding forms in 3D graphic statics have one more dimension than the forces and forms in 2D graphic statics.

By generating or adjusting the polyhedral force geometries, different funicular forms can be provided using 3D graphic statics. The advantage of this form finding algorithm is that the generated structures are always in equilibrium under a given boundary condition. As long as the force diagram is a set of closed polyhedrons, the corresponding form can stay balanced under the action of applied forces. Therefore, when designing a form with given applied loads, architects can divide the force polyhedrons with additional interior faces to achieve complexity while maintaining the form equilibrium.

Machine Learning

Besides the focus on 3D graphic statics, the interest in the application of machine learning to design and optimization has increased considerably (Zheng 2019, Showkatbakhsh, Erdine et al. 2020).

For example, a machine learning model can be trained to classify images of geometries based on the architect’s aesthetic tendency (Turlock and Steinfeld 2019). Examples of the funicular structures are translated into black and white spatially distinguished images then used to train a convolutional neural network (CNN) model. The authors randomly generate a large number of structural models, flatten them into images, and then ask volunteers if they think the images are beautiful. By this method, the trained CNN can learn the aesthetic indicators in the sense of architecture based on the answers of the volunteers. Combined with the traditional structural evaluation indicators, the program can find a solution that is evaluated to be both beautiful and structurally stable.

Furthermore, neural networks can be used to solve the computational problem in the field of structural optimization (Aksöz and Preisinger 2019). The authors use an artificial neural network (ANN) to learn the stress conditions and coping methods in finite element analysis, then use the generated sample data to train the neural network, and finally apply the trained neural network to generate structure solutions based on the stress conditions given by the user. In order to simplify the problem, the authors divide the complex structure system into small units, and the overall structure is composed of each small unit optimized by the ANN. Similarly, the structural computation process is optimized by training ANN models (Yetkin and Sorguç 2019), but the optimization is applied to small truss structures.

Problem Statement and Objectives

In this paper, we aim to answer the following questions: 1) how to integrate the novel strategies in structure and computation and develop a digital process that could translate and optimize the routine structure system into funicular structure as another design option for architects; and 2) how to re-define and optimize the construction module using machine learning techniques.

Therefore, in this research, we propose a geometry-based generative design method to generate and optimize a floor structure with funicular building members. This method challenges the antiquated column system, which has been

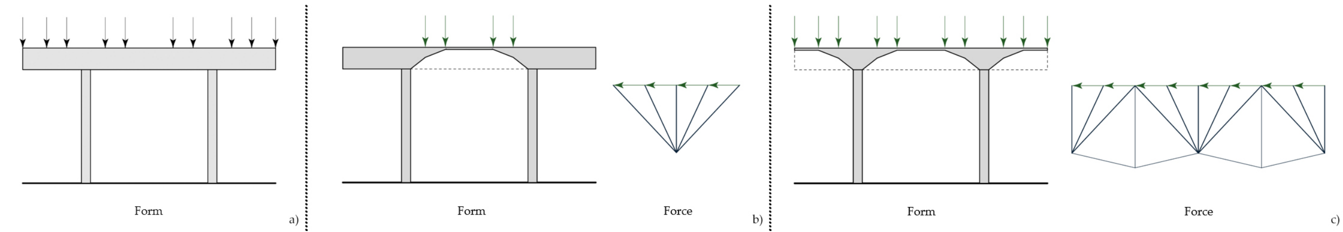

Figure 3. Funicular solution for 2D domino structure. a) flat structure. b) arch structure and its force diagram. c) extendable arch structure and its force diagram.

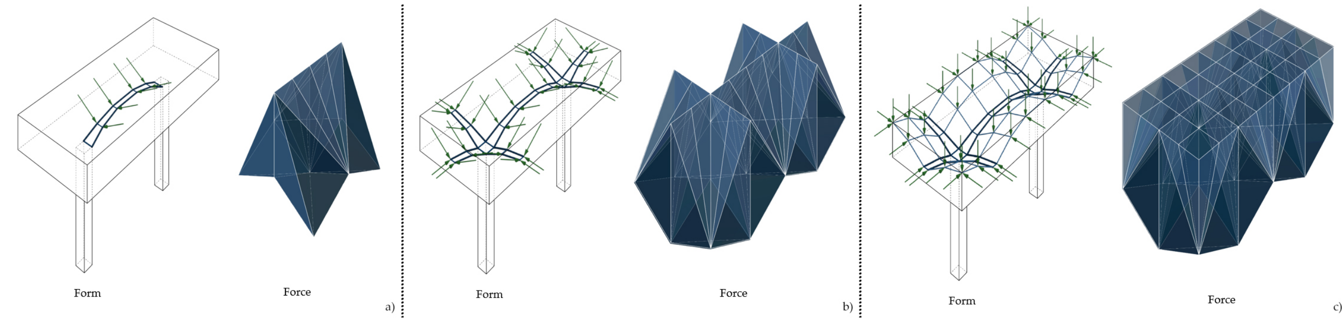

Figure 4. Funicular solution for 3D domino structure. a) flat structure. b) arch structure and its force diagram. c) extendable arch structure and its force diagram.

used for more than a century. By inputting the floor plan with the positions of columns, designers can generate a variety of funicular supporting structures, resulting in more choices for designing a floor structure, rather than only columns and beams. This ability to generate floor structures with funicular forms can foster the development of architectural spaces with more diverse design elements. We further apply machine learning techniques (ANNs) to evaluate and optimize the funicular structure's structural performance and constructability, thus finding the optimal solutions within the almost infinite solution space. To that end, a machine learning model is trained and used as a fast evaluator to help the evolutionary algorithm find the optimal designs. This interdisciplinary method bridges computer science and structural design, providing flexible design choices for generating floor structures.

METHODOLOGY

The Topology and the Subdivision

To re-design a space with the Maison Domino system into funicular forms, the topology should be defined, including the graph relationship (connectivity) between each column. For any floor plans with columns in any position, straight lines can be drawn between pairs of columns, representing the main structural members. Once there is at least one line connecting to each column and no lines are overlapping, the topology is legal, and the complete graph shows the initial structural members.

Figure 3 and Figure 4 explain the logic for constructing forces that could generate a more efficient and ecological funicular solution from a regular Domino module as force and form dual diagrams. Shown in Figure 4, in the funicular alternative, the connection between two columns is re-established with a funicular arch by designing an aggregable 3D force diagram. Aggregating the forces would cause the connections in the form to further expand to boundaries or other supports, which ultimately become the main structural network among the columns. The total applied load, on the other hand, is represented by a horizontal polygon face at the top of the force diagram. Subdividing the total applied load and converging the subdivided faces to different points in the force diagram allows additional load paths to be generated in the form, and through them the applied loads are transferred to the main structures.

Additionally, Figure 5 and Figure 6 show the steps for generating funicular-arch floor structures from 2D layouts. The first step is to identify connections between vertical structural elements and establish the connectivity map. From the connectivity map, one can determine the force boundary that demonstrates how the applied load is distributed to each structural member. The 3D force diagram is then created for each force boundary. The force diagrams can be aggregated to generate the resulting form. Further, as shown in the different topologies for layouts with columns and walls, the geometries of the constraints can also affect the connectivity map and the force distribution.

Regarding the graph as an initial form diagram, the force diagram can be generated using the graphic statics method as a dual geometry (Akbarzadeh, Van Mele et al. 2015).

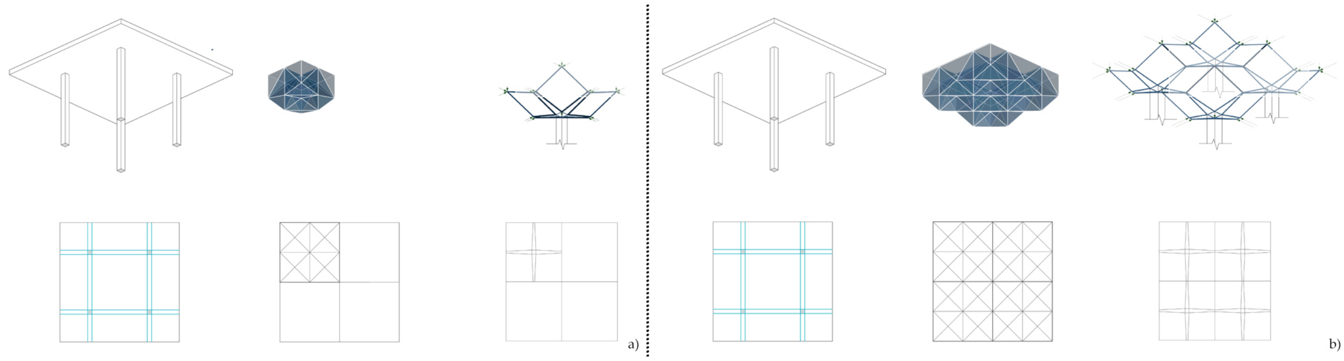

Figure 5. Derive funicular solution from simple layout (with 4 columns).

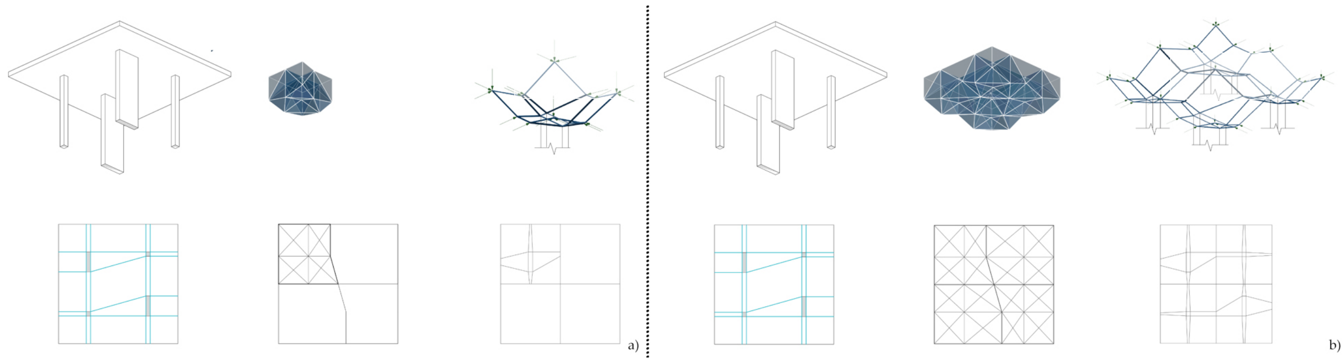

Figure 6. Derive funicular solution from simple layout (with 2 columns and 2 walls).

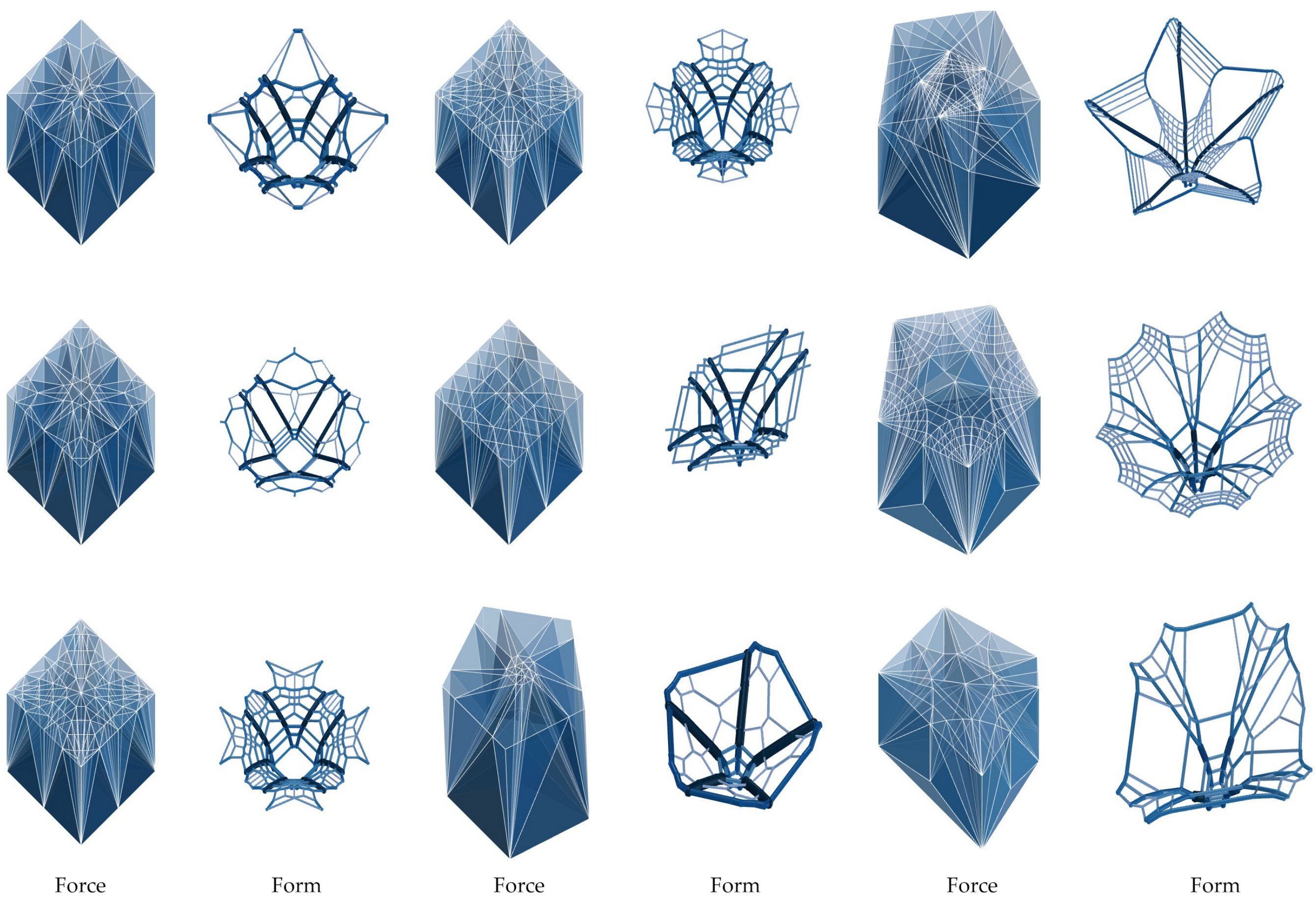

Figure 7. Subdivision exploration.

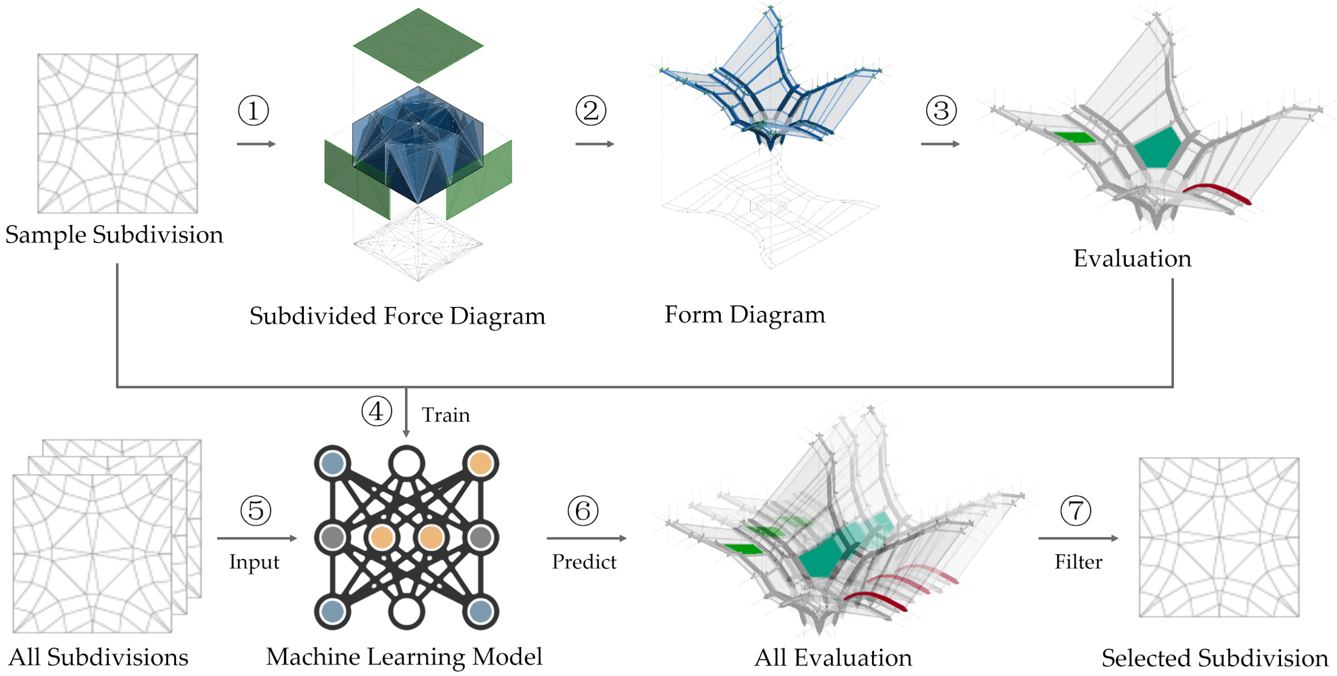

Figure 8. Workflow of machine learning assisted form finding.

The force diagram represents the internal and external force distribution. Subdividing the force diagram divides the internal forces into several sub-forces, thus resulting in internal structural members. With the final subdivided force diagram, the graphic statics method is used again to generate the form diagram as a dual diagram under the boundary constraints of the initial site. Therefore, a funicular arch structure can be designed through this method. With different subdivision strategies, the method can provide designers with highly complex and diverse design options (Figure 7).

Machine Learning Form Finding

There is an almost infinite number of structural forms with different topology and subdivision rules. Optimization is achieved by evaluating each form in terms of its structural performance and material usage. However, traditional optimization methods—for example, enumeration and genetic algorithms (GAs)—usually cause an unacceptable time cost during the process. Therefore, we propose a feed-forward neural network that learns the mapping between the topology using the subdivision rules and the evaluation criteria (Figure 8). By training with a small amount of pre-generated data, the trained neural network gains the ability to predict the structural performance and the material use with high accuracy within milliseconds. Using this neural network model, the form finding process requires less time, finding the best solutions (Zheng, Moosavi et al. 2020).

In detail, the sample subdivision patterns are generated as the training set, in which the parameters to control the subdivision become the input neurons of the neural network. The subdivided form diagrams can be generated according to the subdivided force diagrams and then be evaluated based on their structural properties, which become the output neurons of the neural network. With the sigmoid as the activation function and the mean square error as the loss function, an ANN can be constructed and trained, containing several hidden layers according to the training accuracy.

After training, the neural network becomes an agent with the knowledge to quickly evaluate structural properties. With the trained neural network model, all possible combinations of the subdivision parameters can be inputted for evaluation, and, according to the outputted values, the solutions with the best performance can be filtered, thus finding the best forms.

RESULTS

The Re-design of Space with Random Columns

To test the feasibility of this method, we randomly generate a space with a rectangular floor plan and 11 columns. We choose this example as the case study of using the funicular structure to re-design the architectural space (Figure 9.1). The center points of those 11 columns can be regarded as the connecting points for the topology.

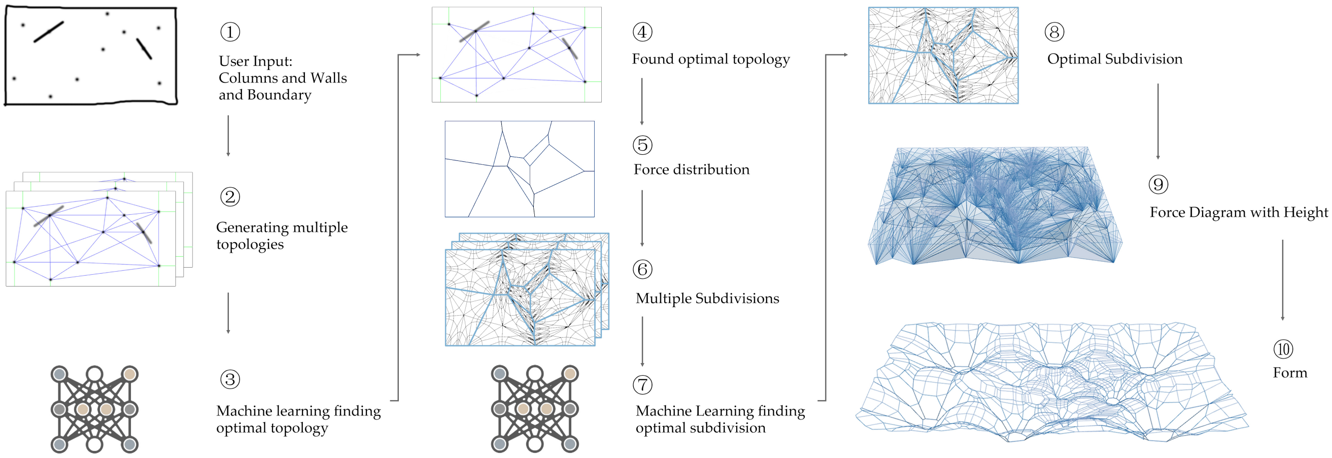

Figure 9. Workflow of generating and optimizing a funicular arch floor structure.

The topology of those connecting points contains 10 fixed lines to the boundary and 30 optional internal lines, which further results in 88 different types of topology. The subdivided force diagrams can be generated by subdividing the 88 force diagrams derived from the topology with a controlling parameter from 0 to 1; the funicular forms can then be created.

Machine Learning Models

In the process of generating the funicular forms, there are 88 types of topology and one parameter with continuous values, which control the generation. Therefore, each form can be represented by a 89-dimension input vector (88 values for the one-hot encoding and 1 value for the subdivision parameter), and the evaluation is a two-dimension output vector showing the values of the two criteria. After the experiment, we found an optimal setting for the hidden layers and the activation and loss functions that results in a high level of accuracy in predicting the structural performance and material use from the topology and the subdivision rules.

In detail, Figure 9 shows the workflow of the machine learning process. After receiving the input settings of the columns and walls from the users, different topological graphs can be generated as sets of parameters representing the initial boundary of each force cell (Figure 9.2). By training a neural network (Figure 9.3) with the method described above, one can find the best topological graph with the smallest average edge length, indicating the most equal distribution of the main forces (Figure 9.4).

With the selected topological graph as the initial boundaries (Figure 9.5), different subdivision rules are applied, resulting in multiple force diagrams (Figure 9.6) and the corresponding form diagrams. Another neural network (Figure 9.7) can be trained and used to find the best force and form diagrams, with the input as the subdivision parameters and the output as the structural properties.

For the output evaluative value, we choose to use the material usage as the criterion. In Figures 10 and 11, we apply some of our most successful subdivision strategies to the simplest layouts and compare the volume of the generated arch floor structure with the regular slab. As shown in the statistics, our generated results save from 30% to 50% materials compared to those obtained through the regular method, but the best material-saving solution is found by the neural network.

Finally, the trained neural network finds the optimal results as the subdivision parameters (Figure 9.8). By transforming the subdivision parameters into the 3D force diagram (Figure 9.9) and its corresponding form solution, the network can generate the best form with the largest material savings (Figure 9.10).

Form Finding Results

After training with the generated samples, the neural network has the ability to act as an evaluation agent to give real-time feedback on the two criteria values. Under the guidance of the neural network model, forms with better structural performance and lower material use are found (Figure 12).

CONCLUSION AND DISCUSSION

Architectural space structured with the Maison Domino system can be re-designed using a funicular structure generated by the graphic statics method. The topology and subdivision rules control the generation of the force diagrams; therefore, the funicular forms are also

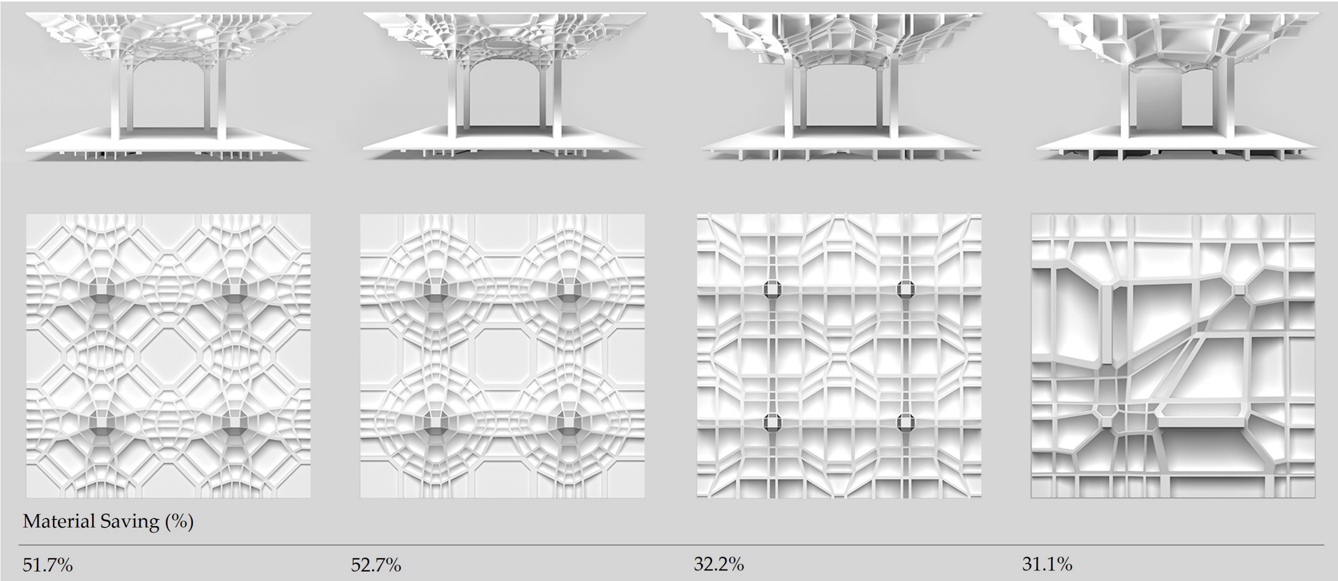

Figure 10. Material saving comparison I.

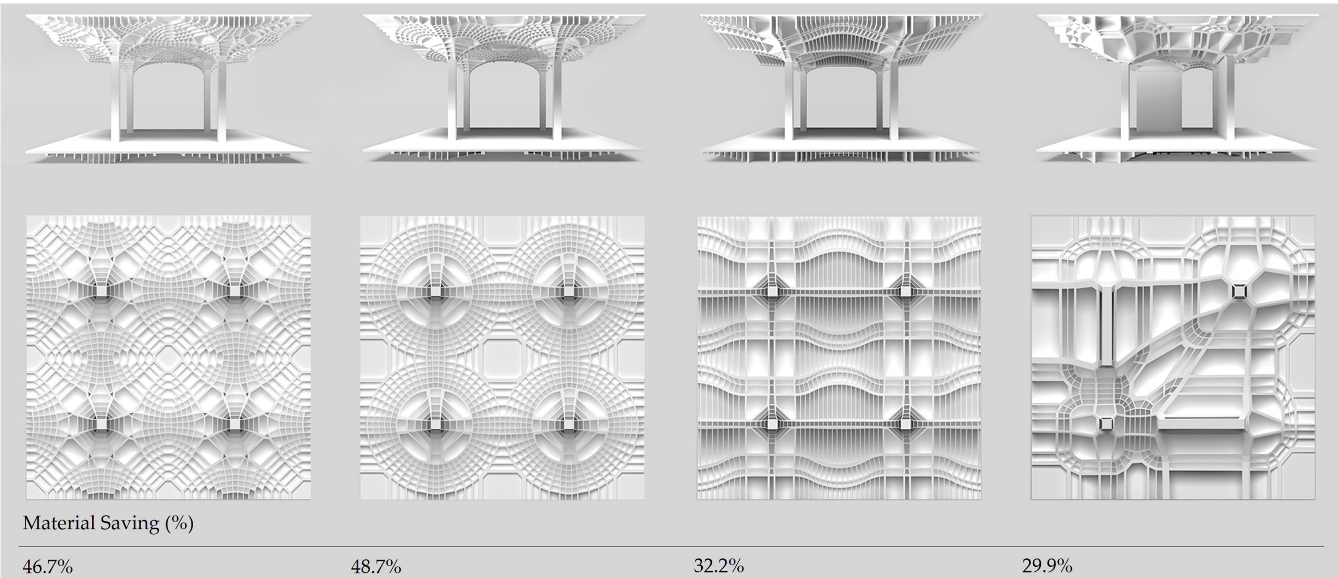

Figure 11. Material saving comparison II.

generated as a dual geometry.

A trained neural network model can find the forms with the user-defined evaluative metrics. The machine learning model is trained and used as a fast evaluator to help the evolutionary algorithm to find the optimal designs. This method spans the interdisciplinary border of computer science and structural design, providing flexible design choices for generating floor structures.

The future research of this project includes improving the topology and subdivision rules, as well as increasing the accuracy and processing speed of the machine learning model.

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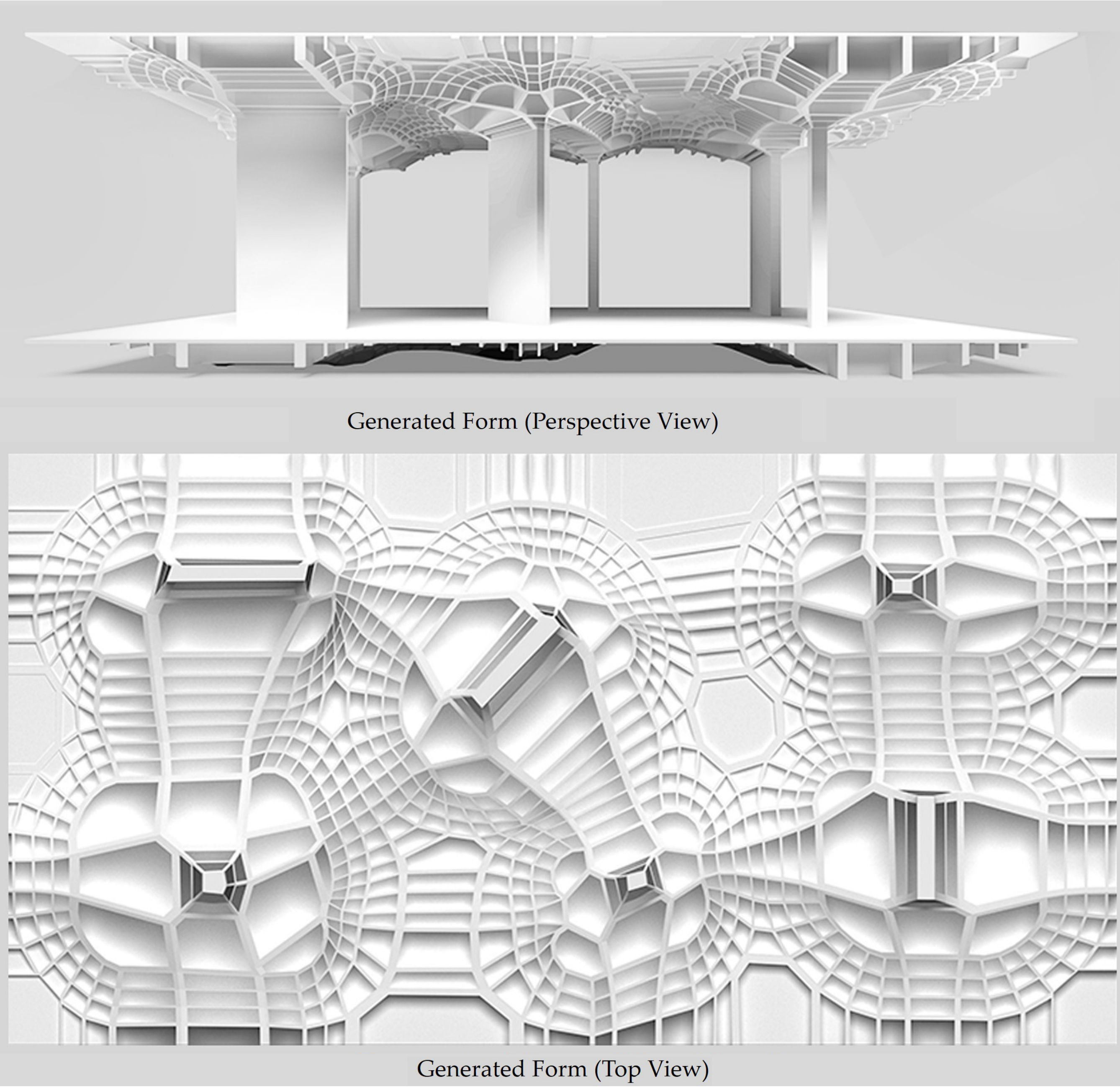

Figure 12. Final form.

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IMAGE CREDITS

Figure 1: from Internet sources.

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