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# Kerf Bending and Zipper in Spatial Timber Tectonics

## A Polyhedral Timber Space Frame System Manufacturable by 3-Axis CNC Milling Machine

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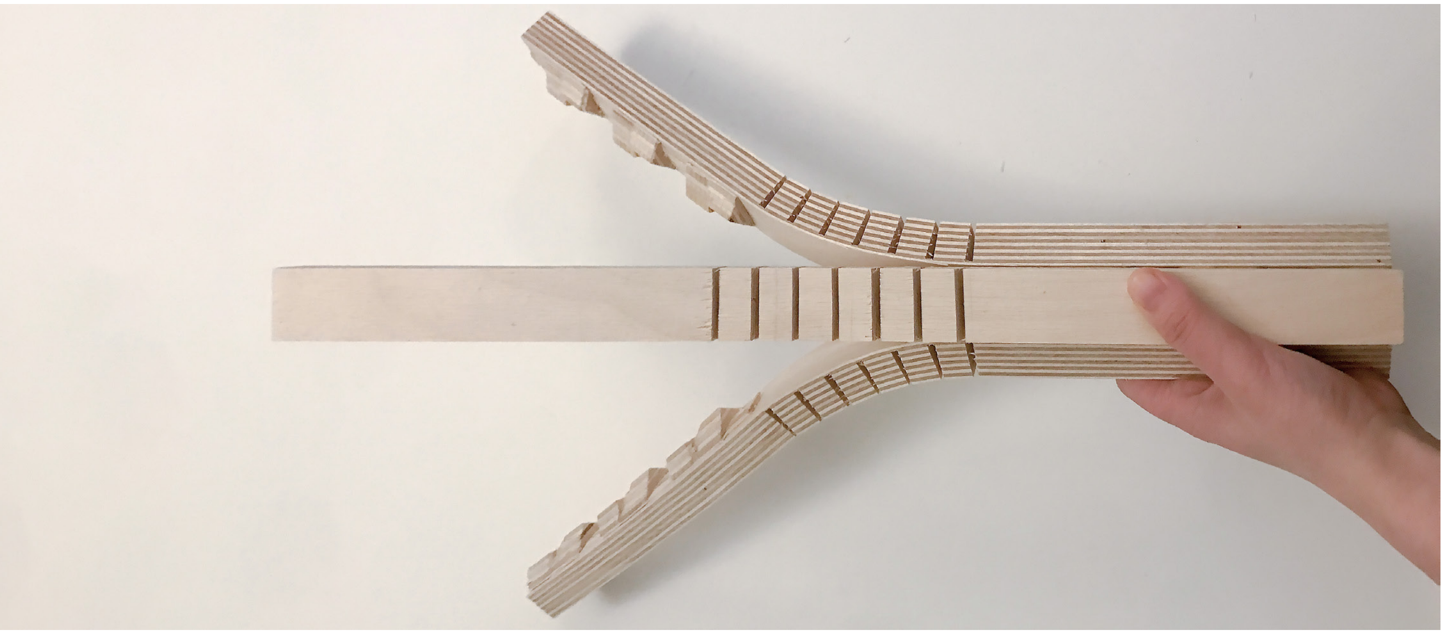

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1 One branch of the bar shows Kerf-bending part and Zippered node

## ABSTRACT

Space frames are widely used in spatial constructions as they are lightweight, rigid, and efficient. However, when it comes to the complex and irregular spaces frames, they can be difficult to fabricate because of the uniqueness of the nodes and bars. This paper presents a novel timber space frame system that can be easily manufactured using 3-axis CNC machines, and therefore increase the ease of the design and construction of complex space frames. The form-finding of the space frame is achieved with the help of polyhedral graphic statics (PGS), and resulted form has inherent planarity which can be harnessed in the materialization of the structure. Inspired by the traditional wood tectonics kerf bending and zippers are applied when devising the connection details. The design approach and computational process of this system are described, and a test fabrication of a single node is made via 3-axis CNC milling and both physically and numerically tested. The structural performance shows its potentials for applications in large-scale spatial structures.

## INTRODUCTION

Space frames are widely used in spatial constructions as they are lightweight, rigid, and efficient. Despite the fact that the systematic development of specialized production methods and digital fabrication technology has enabled the construction of timber structures to reach a new level (Lennartz and Susanne 2015), the fabrication of such spatial structure is still not easy due to the uniqueness of bars and nodes, and always involves robotic fabrication, multi-axis machining, 3D printing as well as casting. The diverse nodes of some branching wooden structural systems are produced by flip-milled on a CNC router (Lamere and Gunadi 2019), and other spatial wooden structures introduce steel sleeve anchor or tension rods to help construct the nodes (Momoeda 2017; Teeple 2015).

To reduce the complexity of the design and fabrication of space structures, this paper presents a novel timber space frame system that blurs the boundary between bars and nodes and can be milled simply by 3-axis CNC milling machines. This system incorporates the traditional manufacturing techniques of kerf bending and dovetailed joints and introduces a new connection using zippered wood. The form-finding process is achieved with the help of polyhedral graphic statics (PGS) as the inherent planarity of the resulted form is required for developing the structural details.

### Kerf-bending and zippered-wood

In practice, there are many techniques to bend a piece of wood into the desired curvature. One way to easily create the precise bending wood surface is the relief-cutting process known as kerfing (Mansoori et al. 2019). It is a well-established carpentry technique for producing curved wooden pieces in a wide range of applications. By cutting a series of deep notches (or kerfs) at the bending area allows for in-plane expansions and compressions perpendicular to the cutting lines (Figure 2). If the two edges of each kerf on the compression side are tightly attached, a mathematical relationship can be found among the angle of bending, the thickness of the wood, and the dimension of cutting slots.

The zippered wood connection was inspired by dovetailed joints which provide interlocking between the components while hiding the connection details. A series of 'pins' cut to extend from the end to interlock with a series of 'tails' from another board (Figure 3). This connection can not only accommodate the orientation change of the bars but also achieve the desired curvature and twisting (Satterfield et al. 2020).


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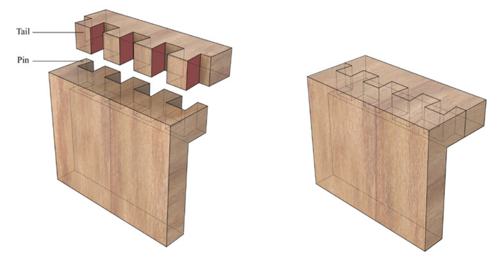


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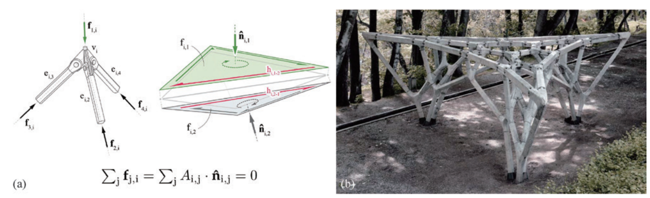


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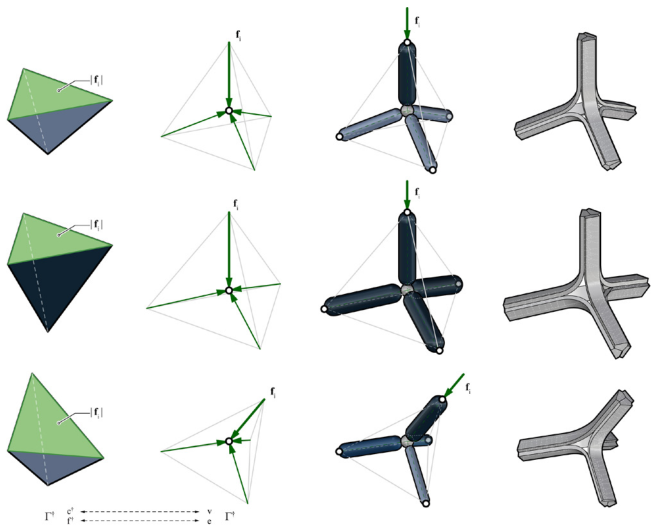
2 Kerf-bending technology allows the rigid wood to bend by the desired angle when the pair of edges of each kerf is tightly attached.

3 Dovetail joint manufactured using classic technology, highlighted are fraction surfaces.

4 (a) a spatial structural joint in equilibrium, and the reciprocal relationship between polyhedral form and force polyhedron (Akbarzadeh et al. 2019); (b) the first built structure designed by using 3D Graphic Statics methods (Akbarzadeh et al. 2017; Bolhassani et al. 2018) (Courtesy of PSL).

### Polyhedral graphic statics

Geometric structural design methods depend on 2D reciprocal diagrams are regarded as a compelling design tool that has long been studied and practiced. The geometric interrelation between force and form was initially proposed by Rankine (1864), and Maxwell (1870) formulated the topological and reciprocal relationship as reciprocal form and force diagrams. The development of 3D graphic statics (3DGS) further increased the ease of designing complex spatial structures. In the realm of 3DGS, the method uses polyhedral reciprocal diagrams, usually referred to as polyhedral graphic statics (PGS), has recently been



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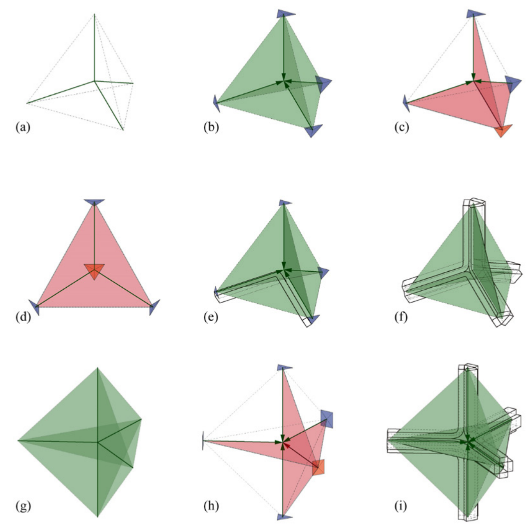


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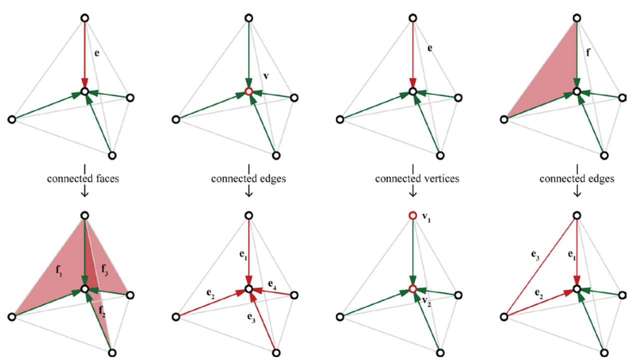


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5 The form can be manipulated by changing the force diagram.

6 Fast queries of geometric and topological information are made possible through the half-face data structure.

7 (a) Spatial node with four bars; (b) edges and faces; (c) triangular section profile for one edge determined by connected three faces; (d) aligned view for the edge section; (e) side face perpendicular to the corresponding input face; (f) all side faces perpendicular to their corresponding input faces; (g) Spatial node with five bars; (h) tetragonal section profile for one edge determined by connected four faces; (i) final generated parts.

developed and extended (Akbarzadeh 2016; Akbarzadeh et al. 2015a; McRobie 2016;). It provides methods to find the equilibrium of structure in 3D by enforcing the closeness of the global force polyhedron and nodal force polyhedrons, and the reciprocity is built by projecting form polyhedrons to force vertices, form faces to force edges, form edges to force faces, form vertices to force polyhedrons, where the force magnitude of each edge in the form polyhedron is equivalent to the area of the corresponding face in the force polyhedron (Figure 4). The inherent planarity of the polyhedral geometries has the potentials to be harnessed for efficient construction processes.

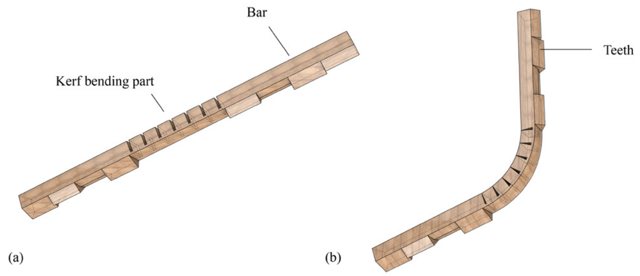
## DESIGN METHODOLOGY

This research is an ongoing investigation in the design and construction of timber spatial structures designed by polyhedral graphic statics. The design approach built upon the reciprocal form and force diagrams of PGS will be explained in this section. The computational model is implemented using Rhino Python.

### Form-finding through PGS

This timber space frame system is developed upon polyhedral forms as the intrinsic planarity helps develop the detail of kerf bending. PGS is used because it allows manipulating the form while being aware of the internal force distribution. PolyFrame (Version 0.1.9.2; Nejur and Akbarzadeh, 2021), an efficient computational PGS plugin for Rhinoceros® (Version 6.0 SR29; Robert McNeel Associates, 2020), together with its underlying half-face data structure (Nejur and Akbarzadeh, 2021) are used throughout the design process.

To start the form-finding process, a set of closed polyhedrons is made as the force diagram, followed by the generation of its corresponding form diagram. The simplest frame with one node is used here to demonstrate the adjustment of the form as well as the organization of the geometrical and topological data (Figure 5). Through the half-face data structure, fast queries of needed information are made possible (Figure 6).

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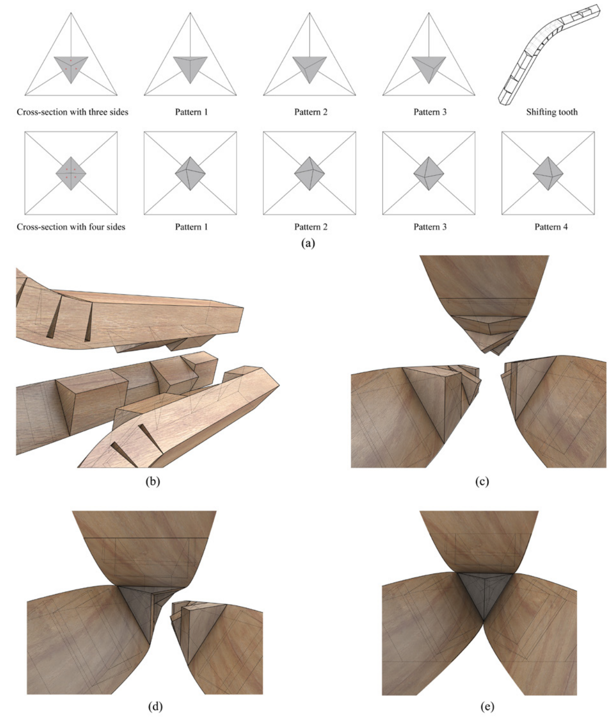


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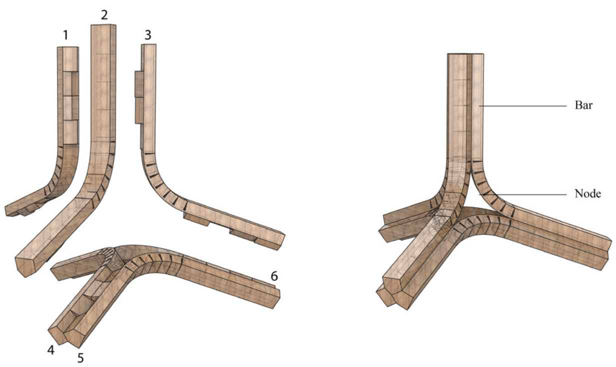


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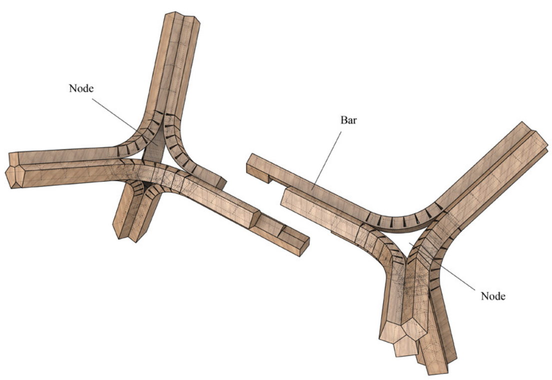


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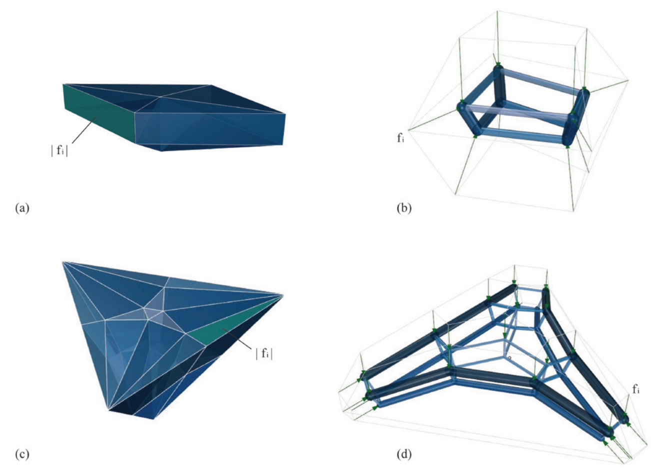


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8 (a) One part of the frame with kerf-bending portion and zipper teeth; (b) the part after bending with the edge pair of each kerf tightly attached.

9 (a)The logic of generating zippered ‘tooth’ and the mode of patterns for triangle and quad-rangle cross-section; (b)(c)(d)(e) details for the three-directional zippered part.

10 The assembly sequence.

11 The connection between two neighboring joints.

12 (a) The force diagram of a table; (b) the form diagram of the table; (c) the force diagram of a shell; (d) the form diagram of the shell.

## Materialization

After getting the form diagram (Figure 7a), the edges are materialized into timber bars. The cross-section of each bar is determined by both the connected faces of the corresponding edge and the magnitude of its force. Taking one edge from the form diagram, the number of sides of the profile is determined by the count of the connected faces; the direction of each side is determined by the normal of the corresponding face; the area of the profile is proportional to the internal force of the edge.  For instance, if the edge connects to 3 faces, the profile will be triangular (Figure 7b ~ 7d,); if the edge connects to 4 faces, the profile will be quadrangular (Figure 7g~i).

Later, a smooth singly curved blend is generated for each pair of bar sides that are connected and sharing the

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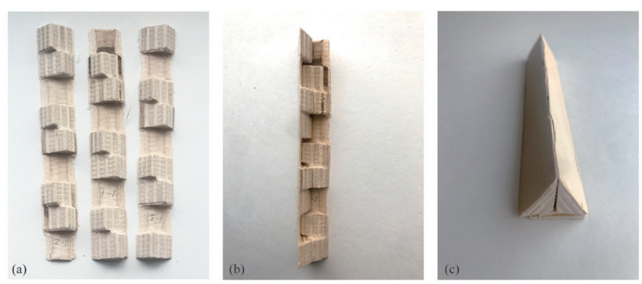
same face, forming the kerf bending node that joins all the connected bars (Figure 7e). Then, the side extrusions of each bar as well as the zipper teeth are made along the corresponding edge, whose details will be described later. Finally, all the geometries that are associated with each face will be grouped and fabricated as one part. As a result, this simple spatial structure with four bars and one node can be made by only six parts (Figure 7f). Since both the extrusion and kerf bending portions of each part are perpendicular to the same face, it can be unrolled and fabricated from flat material (Figure 8).

The zippered teeth are located on the interior side of the straight segment of each part, serving as the mechanism that interlocks all side parts that compose each bar.

As a further description of the interlocking zipper teeth, the generation of the tooth patterns for one bar is illustrated as follows (Figure 9). First, the length of the bar is evenly divided into a number of segments. Then, the profile of the bar is split into triangles by connecting its centroid to all corners. Next, the centroid of each triangle is successively used as the new splitting point for the tooth pattern of the

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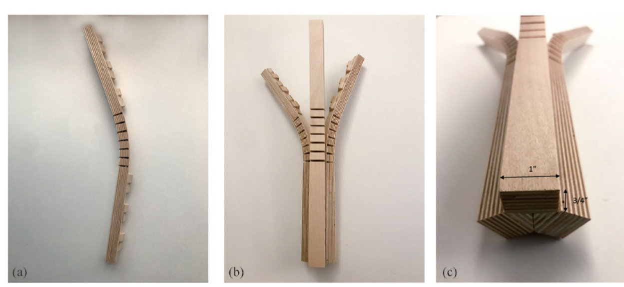


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13 Renderings of a funicular shell and a desk.

14 Assembly process of the teeth of a bar.

15 (a) one part; (b) three parts after assembly; (c) cross-section of a bar.

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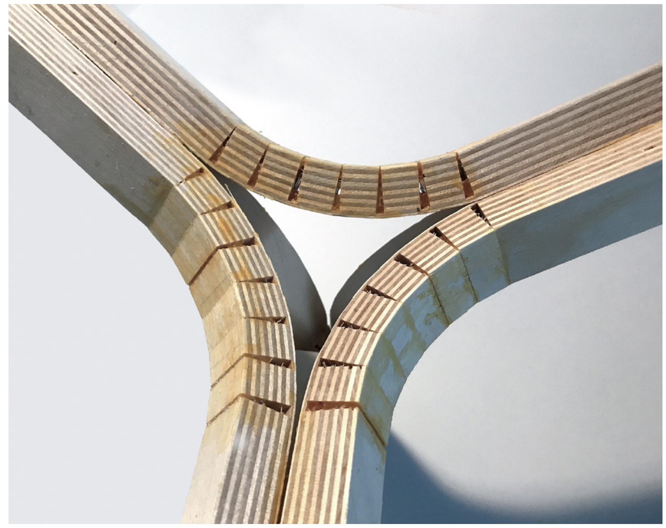

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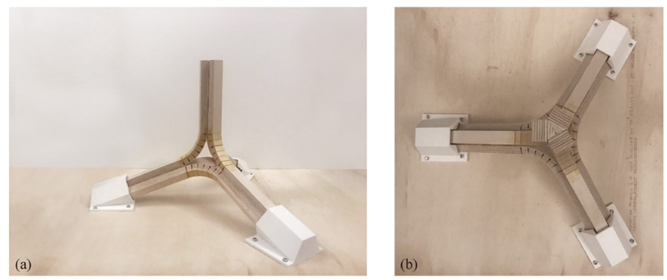


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16 One whole joint after assembly.

17 Details of the kerf-bending part.

18 (a) Set-up for load test (b) top view of the set-up.

segments, and consequently, the interlocking teeth are created.

For kerf cuttings, the cutting depth, cutting width, and kerf count are the 3 key parameters that determine the curvature of the bending. They need to be calculated accordingly such that the 2 exterior edges of each kerf are tightly pressed together after bending (Figure 9b).

The material properties and fabrication constraints are also considered parallel alongside the geometrical development of the space frame system. Timber is selected as the construction material because of its sustainability and ease of processing. All geometries of the structural parts are generated in the way that only the table saw and 3-axis CNC milling machine are needed for rapid fabrication, where the table saw cuts the kerfs and the 3-axis CNC milling machine carves the teeth. It's also manufacturable by a 3-axis CNC milling machine only with flip milling technique. The assembly of the frame is illustrated in Figure 10, and a bonding agent is required on the interface between the parts.

### The application in multi-node polyhedral forms

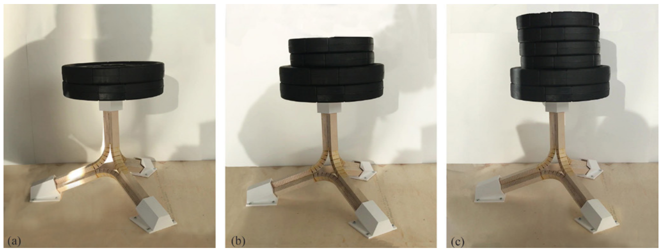
With a connection devised between every two neighboring nodes, the workflow described above can also be applied to complex polyhedral space frames with more bars and nodes. As shown in Figure 11, the two adjacent nodes share the same bar. By varying the contact location of the sides, an interlock is created between these nodes, and the contact area is increased. As case studies, a compression-only funicular shell and a table are generated using the workflow proposed above (Figure 12~13).

## FABRICATION AND EXPERIMENTS

The strength and stiffness of the proposed system need to be carefully investigated as the bending part is made vulnerable due to the kerfs. To evaluate its structural performance, a frame prototype with 4 bars and 1 node is fabricated, assembled, and tested through both physical and numerical methods. The overall fabrication process would use the 3-axis CNC milling machine for the zippered shape and table saw for cutting the wood.

### Fabrication

The simple frame is built within the 450x400x360 mm bounding box, and 3/4" thick plywood is used.

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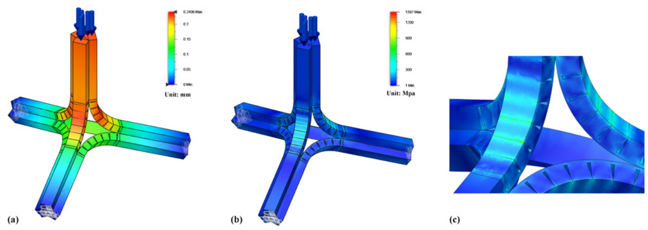


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19 Physical load test for the prototype; (a) 50 pounds of applied load (b) 70 pounds of applied load (c) 90 pounds of applied load.

20 (a) Displacement map; (b) stress distribution map; (c) detail of maximum stress point after bearing the force of 1KN.

Although each part is manufacturable as a whole, the zipper teeth of each part are separately made and glued back in this prototype for a faster fabrication (Figure 14, Figure 15a). For the kerf-bending part, the slots are cut by a 1/8" table saw. For the zipper teeth, appropriate tolerances need to be left to ensure the tight interlocking. Figure 15 (a) shows one part of the assembly, and Figure 15 (b)(c) show one assembled bar by interlocking and bonding three parts together. As explained before, all the exterior edges of each kerf are pressed together after the assembly, and bonding agents are applied to fix the curvature and strengthen the structure (Figure 16~17).

### Physical test

Due to the limited accessibility of testing machines, a simple loading test is conducted to study the strength and stiffness of the prototype. The supports are 3d-printed and screwed on a wood panel to confine the 3 bottom bars from any lateral movement (Figure 18). The self-weight of this prototype is 857g, and after applying a total load of 90 pounds on the top bar, there is no visible buckling or deformation, which shows the potentials of its load-bearing capacity (Figure 19). To further understand its structural performance and limit, a universal testing machine will be used for the next step to complete an accurate experiment.

### Numerical simulation

The Autodesk Fusion 360 is used to simulate the stress distribution and displacement of the prototype under heavier loads. A total load of 1 KN is applied to the top bar while the bottom 3 bars are fixed at the end. The result shows stress concentrating on the exterior side of the kerfs, where the maximum stress is 34.93 MPa and the maximum displacement is 0.006mm (Figure 20).

## CONCLUSION

This paper introduces a novel timber space frame system by combining kerf bending with zippered wood, which can be easily manufactured by accessible tools like 3-axis CNC milling machines. The assembly is also made easy since no jig or locator is needed. This system greatly facilitates the design and fabrication of complex timber space frames and allows for rapid fabrication and construction. A computational pipeline is also developed based on PGS for fast modeling and user-friendly parameter control.

The outcome of both the physical test and numerical simulation show promising structural performance in terms of the little deformation under the heavy loads. It also contributes towards the applications of polyhedral graphic statics in materialization and construction.

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