Course Title:

Geometry-based Structural Design

Material and Detail Computation for Structural Geometry

Fall 2025
ARCH 7320
Tuesdays 1:45pm - 4:45pm
Meeting location: MEYH B07
Course outline

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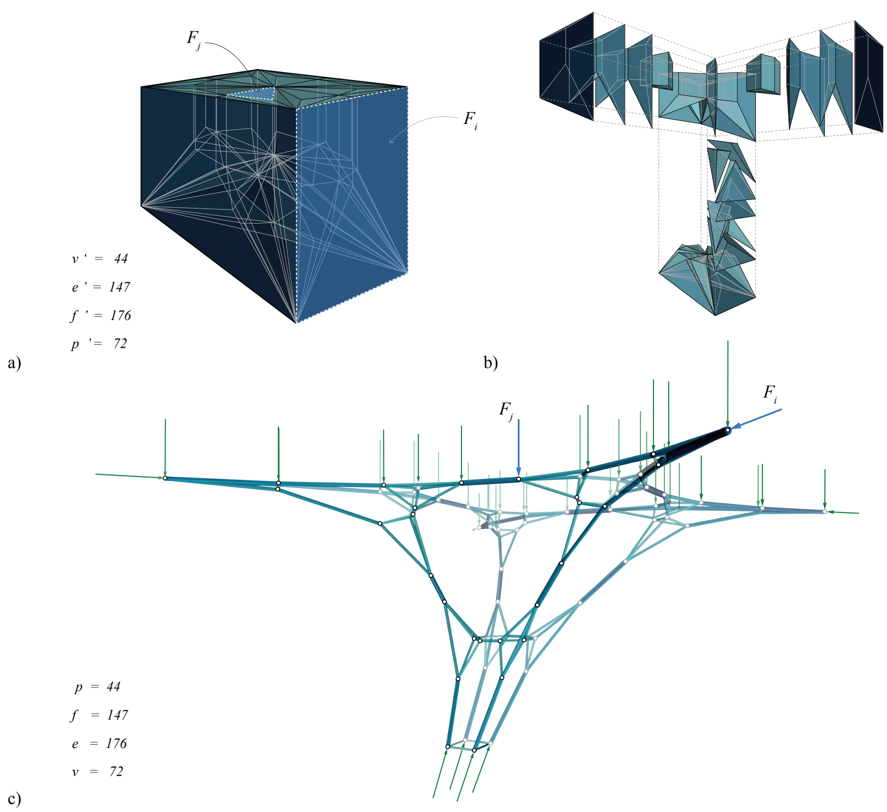

Figure 1: (a) Force diagram consisting of convex polyhedral cells; (b) the exploded axon of the force cells; and (c) the corresponding spatial funicular structural form.

1 Course description

Material Computation for Structural Geometry course provides a comprehensive introduction to novel geometric methods of structural design based on 2D and 3D graphical statics (Rankine, 1864; Maxwell, 1870; Wolfe, 1921; Akbarzadeh, 2016). The primary emphasis of the course will be on (i) developing a general understanding of the geometric representation of their internal and external equilibrium forces; and (ii) designing material tectonics based on the flow of forces in the system. Considering both force flow and material methods are necessary for designing efficient and innovative architectural structures. This semester, special consideration will be given to material and computational methods for the detail design of joinery and assembly process of spatial node. An appropriate fabrication techniques needs to be studied to construct the entire complex geometry of the structure.

Note that this course is based on ongoing research in the field of 3D graphical statics, and therefore provides students with the opportunity to directly contribute to the current research in geometric methods of the structural design. Familiarity with the parametric environment of Grasshopper is required, and code-writing ability is an asset. Particular attention will be given to structural model making and precise structural, architectural and assembly drawings. The outcomes of the course will become a primary collection of Polyhedral Structures Laboratory. Also, a unique summer research fellowship will be available for highly motivated students to build a one-to-one scale structural prototype based on the structural systems developed in the class.

2 Methodology

The course is divided into five consecutive parts with specific intentions; *Part I* will introduce the geometric principles of equilibrium of structural forms (Wolfe, 1921; Akbarzadeh et al., 2015a); *Part II* will focus on structural form finding using geometric techniques (Akbarzadeh et al., 2015c; Lee et al., 2016); *Part III* will concentrate in manipulating the geometry of the structural form and its force diagram to explore various architectural schemes (Akbarzadeh et al., 2015b; Akbarzadeh, 2016); and *Part VI* will specifically emphasize the choice of material and the fabrication techniques to construct complex spatial forms. Table 1 provides a brief overview for each part and its relevant aims and objectives.

2.1 Aims

Therefore the course has the following particular intentions:

- to introduce the concept of equilibrium using geometric techniques, expanding the reciprocal relationship between the elements of an equilibrated structural form and its force diagram;
- to emphasize the use of geometry in designing complex yet efficient structural forms and deriving the internal and external forces using geometric diagrams;

Course sections	*Aims*	*Objectives*
Part I: the principles of equilibrium	• introducing geometric equilibrium of forces in 2D and 3D; • hinting on the properties of form and force diagrams; • defining global and nodal equilibrium;	• to construct parametric form and force diagrams; • to represent different states of equilibrium geometrically;
Part II: form finding explorations	• introducing the technique aggregation in the force diagram; • describing the force subdivision technique; • providing the computational framework for form finding;	• to construct complex structural forms by designing their force diagram; • to produce various structural forms by redistributing the internal forces;
Part III: manipulating/articulating the design	• introducing geometric degrees of freedom of the form and force diagrams;	• iterative design process to fulfill specific architectural needs;
Part VI: materializing the structural geometry	• introducing various materials translating structural geometry into building components; • reviewing multiple fabrication techniques for constructing spatial structural geometry;	• to choose a specific material and translate the structural geometry into a volumetric object based on the properties of the chosen material; • to devise a proper fabrication technique for constructing the structural geometry;

Table 1: Various parts of the course and their aims and objectives.

- to simplify the understanding of complex structural concepts using geometric language instead of numerical methods; and,
- to investigate different materials and fabrication techniques to realize spatial structural forms .

2.2 Objectives

On completion of this course, students should be able to:

- describe the equilibrium of structural concepts using geometric methods of graphical statics in 2D and 3D;
- construct structurally informed, novel architectural concepts and derive the

internal and external forces in the system geometrically; and,

- understand the challenges in materializing spatial structural forms and develop appropriate fabrication techniques to construct their complex components (Jeska and Pascha, 2014; Lennartz and Jacob-Freitag, 2015; Weinand, 2016).

3 Grading

Table 2 provides the grading criteria of the course:

Assignment	*% of grade*
Session attendance	10
Exercises	30
Physical Prototype	40
Final structural model	20
Force model	10*

Table 2: Grading criteria;
∗ Additional points for the groups who build their force model for their final projects.

4 Lecture schedule

Table 3 represents the schedule as well as the tiles of the lectures of the course and their related exercises.

5 An Attendance Policy

The Weitzman Course Attendance Policy can be found in the Weitzman Student Handbook, but it is very general. It states, ”Policies regarding absence from classes are determined by the instructor(s) responsible for the course.”

6 Pre-requisite information

This course does not have any prerequisites.

Session Date	*Week*	*Topics*
Aug 26	w1	Introduction to geometric methods of structural design
Sep 2	w2	Exercise on a parametric node and its force equilibrium
Sep 9	w3	Compression-only form finding: force subdivision and aggregation
Sep 16	w4	Exercise on compression-only form finding
Sep 23	w5	Compression-and-tension combined systems
Sep 30	w6	Exercise on combined system of forces
Oct 14	w7	Convex hull, Extended Gaussian Image, and the data structure of a node
Oct 21	w8	Mid-term review: physical, structural model and proposition for the building block
Oct 28	w9	Material geometry: computational design workshop
Nov 4	w10	Exercise on section development
Nov 11	w11	Structural performance assessment
Nov 18	w12	Exercise on structural analysis
Dec 2	w13	Assembly techniques and strategies and Exercise on the assembly and part-to-whole relationship
Dec 16	w14	Final review

Table 3: Lecture schedule with specific dates for exercises, midterm, and final review dates.

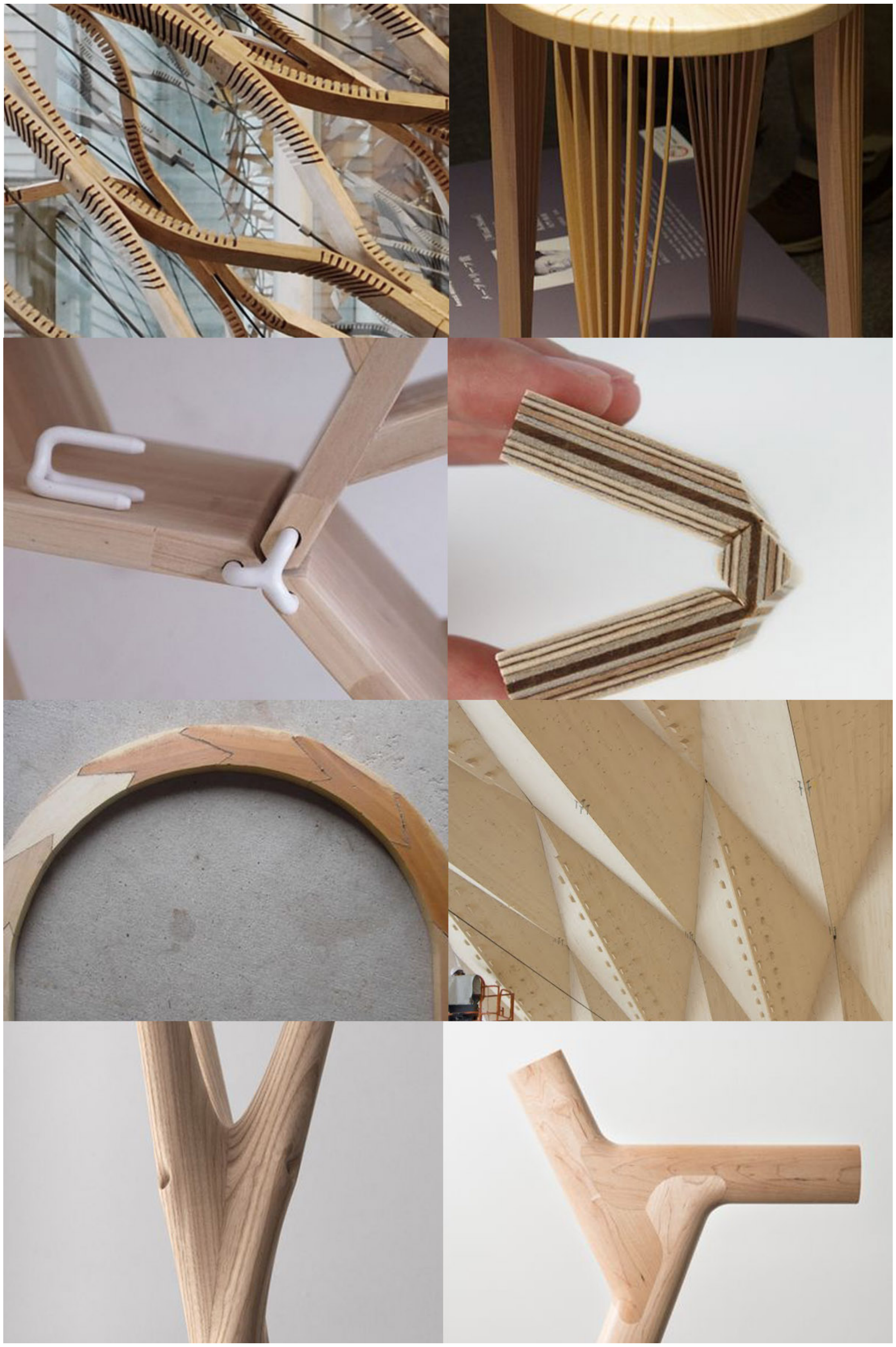

Figure 2: Various joint detailing and fabrication techniques applicable to structural timber.

7 Exercise I

Construct a parametric model of a node with four bars with all its connected faces. Construct the force diagram for this node using the procedures explained in Figures 1.14, and 1.18 of the hand out. Construct the force diagram in a parametric environment and show how controlling the form's geometry results in various geometries of the force diagram for compression and tension-combined cases.

- The user should be able to move the form's vertices and receive the resulting force diagram.
- The diameter of the form edges should be updated based on the areas of the force diagram's faces.
- Vectors should represent the internal forces, and the vectors' direction should match the normal direction in the face. The edges in tension should be represented by red and the ones in compression with blue color.

7.1 Deliverable

The following items should be delivered for the next session of our class:

- A working parametric model of the node should be delivered.
- a short video showing the change in the geometry of the form and its updated force diagram.
- The model should particularly exhibit the qualities of the force diagram for compression-and-tension combined systems.
- The vectors should be color-coded, i.e., the applied load vector should be green, blue for compression force, and red for tension in the members.
- for both diagrams show faces and cells.
- the edge vectors and the direction of faces should be constructed and presented in the final model.

8 Exercise 2

The objective of this exercise is to reinforce the understanding of the form and force diagram using a procedural approach of graphical statics and parametric modeling. The emphasis will be on 2D graphical form finding, and the intention is to explore multiple design solutions using the subdivision approach.

8.1 Design problem

Let's design a bridge that spans 100ft/30m from point A to point B. The height of the deck of the bridge should be at least 5m/15ft above the height of the support A and support B, which is 5 meters lower than support A in elevation. The design should show a simple design as well as the same design subdivided by choosing a method of subdivision explored in advance of the construction of the bridge.

8.2 Deliverable

The following items should be delivered on Sept 16th:

- the complete form and force diagrams for the constrained model of your bridge design. The Rhino/Grasshopper file should be clean and precise. Try to control your parameters so that changing one parameter does not result in representational failures.
- the preferable format for your drawings would be .gh/.3dm files.
- consider specific material or method of construction for your bridge, such as stone, concrete, brick, steel, and design the volume of your bridge accordingly.
- presents a rendering of your bridge merged with the landscape around it.

9 Exercise III

Design a compression/tension only structural network to support a modular bridge spanning 160 meters (1”:10m in the model space)

- This exercise has both digital and physical
- The concept should be constructed in Grasshopper and be constrained to specific support locations as specified in the Rhino file shared with you. It should be demonstrated that a change in the location of the supports results in a new equilibrium form geometry.
- each team should explore various subdivision ideas and extract the final structural concept based on those explorations; the various subdivision techniques and their relevant steps should be explained in drawings;
- It is recommended that each team devise its own subdivision techniques and the process of using them to reach the final concept. I.e., the same reproduction of the examples of the class is not allowed.
- the concept should convey the possibility of its use as a modular system; therefore, the connection configuration of the adjacent modules might be considered as a design criterion;
- the maximum length for the compressive members, which are not connected to the supports, should not exceed 3 inches in the model.

9.1 deliverables (due Sept 30th)

The structural concept should be delivered in **all** following formats:

- *drawing* (due Jan 30): the drawings of the form and force diagrams should be presented in drawings similar to the format of the shared sample drawings. The drawing sizes should not exceed the 8.5 ” x 11” document size.

  The digital format should also be constructed in Adobe Illustrator, and special attention should be given to the line weights of the drawing; members with a greater force magnitude should be presented by thicker line weights than those with a lesser magnitude of the internal force. The color convention of the reaction/applied forces should be similar to the given examples. You can directly export the pipe diagram drawings from Rhino and overlay them with your line drawings in Adobe Illustrator. The drawings should very well explain the design process as well as the subdivision logic.
- *Parametric model*: The parametric model (*.gh*/*.ggb* format) should be presented parallel with the drawings; it should demonstrate the constraint of the form and force diagram and the level of control in the design by changing the location of the supports.
- *physical model* (due Sep 30th): the physical model should be constructed as a working tension-only structural form constructed from (possibly metal) strands subjected to the applied loads. The model should be constructed on

a piece of plywood (1/4” thickness, 11.25”h x 20”w) veneered with white paper.

10 Exercise 4

10.1 Exercise Objective

This exercise aims to develop a design proposition that will inform the materialization of a compression-only shell to be designed in the following exercise. Building upon the precedents and references discussed throughout the course, you are expected to develop the following components of your design investigation.

Research and Documentation Conduct research on your chosen material and the corresponding construction method. Present your findings in a PowerPoint presentation that includes both historical and contemporary precedents of similar construction techniques. Identify the material and fabrication constraints that will influence your subsequent design development. All references, including academic papers and relevant precedents, must be properly cited.

Material and Construction Strategy Select a specific material and define your intended construction approach. For example, you may choose to work with timber using sheet-based components that incorporate predefined cross-sectional geometries, which can later be adapted according to the flow of internal forces.

You are encouraged to explore and combine various fabrication approaches—additive, subtractive, sheet-based, member-based, or hybrid methods—depending on your design intent and the available resources.

10.2 Deliverable

- Provide a detailed description of the construction strategy for your selected approach, focusing on a specific joint, edge, face, or cell within a spatial structure.
- Propose an appropriate fabrication technique based on the available equipment (for example, a 3-axis milling machine), ensuring that your design adheres to the operational and geometric constraints of the chosen machine.

11 Exercise 5

This assignment explores the application of computational tools in designing articulated funicular structural forms in 3D. The *PolyFrame 2* plugin for Rhinoceros will be used as a computational tool developed by the Polyhedral Structures Laboratory.

Tools

To fulfill the parts of the assignment **5**, you may need a computational framework for your design tasks using 3D graphic statics. Please download the latest version of PolyFrame from the following address:

https://www.food4rhino.com/en/app/polyframe-2

11.1 Exercise parts

This exercise consists of the following sections:

- the exercise starts with the example of a column whose form and force are shared with you. This structure will be used as an inspiration for your further studies in this exercise. You can analyze the structure by understanding the logic of subdivision in its force diagram.
- Each team should come up with a design logic for the subdivision of the force diagram and show multiple steps of the subdivision resolution and its effect on the geometry of the form. For further readings, you can also visit our recent publication by Zheng et al. (2020)

11.2 Deliverable

The following items need to be presented on Tuesday, October 21th:

- The drawings of the form and force diagrams should be presented based on the format of Figure 3. Please pay attention to the presentation's quality and precision of your drawings since it is part of the exercise's grade.
- The drawings of the form and force diagrams of your columns structure constrained to the given boundaries should be presented. Use the drawings of the book as a guide for the style of the drawings. Usually, I get the rhino screen from the color coded diagram and put it under the 2D drawings extracted from the 3D model. Figure 3 shows proper protocol for the line weights and overlaid drawings on the form and force diagrams. The Rhino file of the form and force should also be uploaded to the Canvas.
- We will also revisit your updated precedents and your proposal for fabrication and the materialization of your structure.

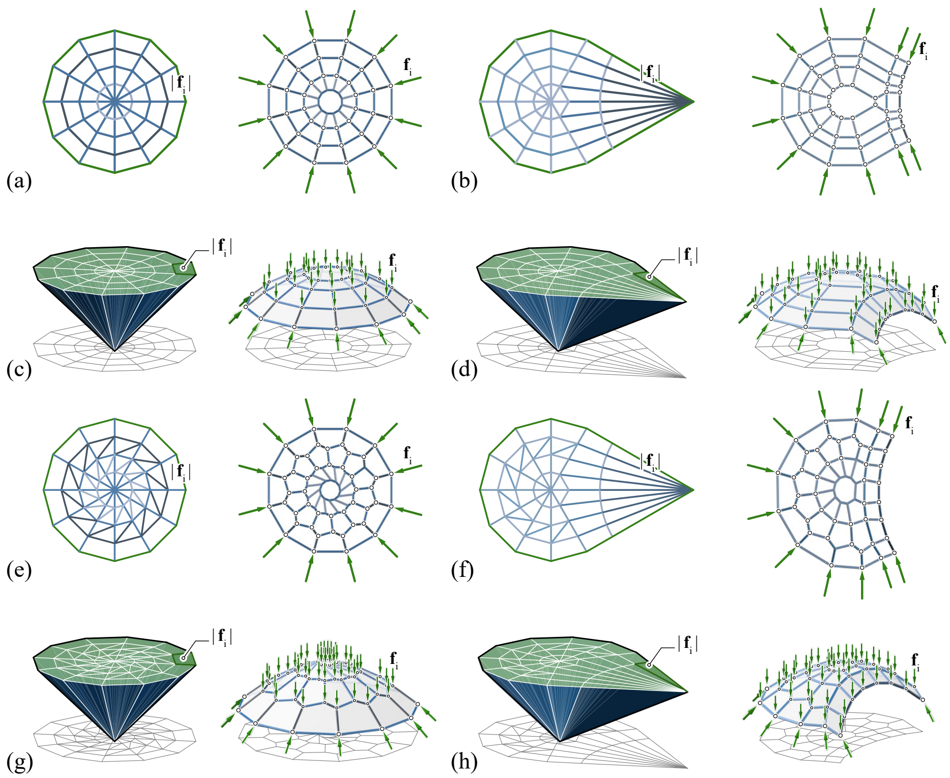

Figure 3: Various 2D (a, b, e, and f) and 3D shell designs by subdividing the force diagram and controlling particular features in the funicular form (c, d, g, and h).

12 Exercise 6: Mid-term Exam

The primary objective of this exercise is to propose a construction technique that translates the funicular design network into a material composition, either a single or a hybrid system. It would be essential to identify the constraints of machining and the limitations of material, and offer design methods to address them.

12.1 List of Deliverable

The following items are necessary for the mid-term presentation.

- A presentation file in the format of PowerPoint including the following sections: (i) introduction, expanding on the type of construction material and precedents that incorporated the materials in the design of building systems; (ii) form-finding exercise and the description of the problem to span 10 x 10 meter columns; (iii) the constraints of the machine or materials; (iv) the proposed technique to materialize the funicular network.
- The physical prototype of the construction and fabrication logic method that identifies the construction challenges/opportunities for the final structure by the end of the semester.

- The documentation of the entire work in the form of a PDF file in the following format: (1) use a letter-size document in its portrait form; (2) consider a 1″ margin from each side and include two vertical columns for the text with a gutter size of 1/4″.

13 Final Review Requirements

The final review material includes a presentation and a physical prototype.

13.1 Presentation and Physical Prototype (50 points)

The following items need to be presented in the final review session in the form of a PowerPoint presentation:

- Introduce the idea behind your strategy in design and fabrication by showing relevant precedents and inspirations;
- Present the novelty of your work by explaining your design and fabrication process;
- Show the renderings of a single component in multiple angles in the same scene settings and lighting shared with you for the mid-term review;
- The details of the connection and the assembly sequence should be shown in a step-by-step animation;
- The relationship between the change in the cross-section of the node and the flow of internal forces. This change can be demonstrated by applying the design to the parametric node in Exercise 1. Thus, changes in the members' angles will alter the design's cross-sectional depth. Otherwise, apply the code to a node with various internal forces.
- The connection of two adjacent nodes;
- Present the overall design approach by explaining various parts of the node and their topological relationship so that a reviewer who is not familiar with the relationship of the parts can understand your approach;
- Break down the code into its constructive modules, functions, or classes, and the relevant explanation of the tasks and output of each module to complete the detailing of the node;
- Apply your design to your chosen funicular polyhedron and show how the forces can be properly transferred to the ground.
- Highlight the novelty of the work in comparison to the studied precedents.
- Present the physical model of your design in a minimum size of a 1.0m (3′) x 1m (3′) structural system to show the details of the fabrication and the overall system assembly.

13.2 Documentation (50 points)

The following items need to be delivered on December 20th:

1. The PowerPoint presentation of the final design and fabrication, including the documentation of the physical model.

2. A functioning code (in Python or a single Python component in Grasshopper) that can run and receive the geometry of a node to generate the detailed design (due Dec. 20th);
3. A 3d model (.3dm file) of a single node, two adjacent nodes, and the entire structure, including the input geometry (due Dec. 20th);

References

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